

SENSITIVITY OF CYCLONES TO INITIAL CONDITIONS: A NUMERICAL APPROACH THROUGH POTENTIAL VORTICITY INVERSION

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INTRODUCTION- Lecture 2

HEAVY RAIN PRODUCING WESTERN MEDITERRANEAN CYCLONE

FACTORS → Two embedded upper level disturbances (positive PV anomalies)
(**dynamical** factors)

How can the internal features of the flow dynamics (jet streaks, troughs, fronts, etc...) present in the initial conditions be **switched on / off** without compromising the delicate 3-D dynamical balances that govern both the model and actual meteorological fields ???



PIECEWISE PV INVERSION

FUNDAMENTALS PV- QG framework

1) Our starting point are the quasigeostrophic equations (in pressure coordinates):

* **Thermodynamic equation (adiabatic)** $\frac{\partial T}{\partial t} = -\vec{V}_g \cdot \vec{\nabla} T + \frac{p}{R_d} \sigma \omega$ $\xrightarrow[\text{Hydrostatic equation}]{T = -\frac{p}{R_d} \frac{\partial \phi}{\partial p}}$ $\frac{\partial}{\partial t} \frac{\partial \phi}{\partial p} = -\vec{V}_g \cdot \vec{\nabla} \frac{\partial \phi}{\partial p} - \sigma \omega$

where for the static stability parameter we will assume: $\sigma = -\frac{\alpha}{\theta} \frac{\partial \theta}{\partial p} \approx \sigma_{ref}(p)$

and remember the expression for the geostrophic wind: $\vec{V}_g = \frac{1}{f_0} \hat{k} \times \vec{\nabla} \phi$

* **Vorticity equation (frictionless)** $\frac{\partial \zeta_g}{\partial t} = -\vec{V}_g \cdot \vec{\nabla} (\zeta_g + f) - f_0 D$ $\xrightarrow[\text{Continuity equation}]{D = -\frac{\partial \omega}{\partial p}}$ $\frac{\partial \zeta_g}{\partial t} = -\vec{V}_g \cdot \vec{\nabla} (\zeta_g + f) + f_0 \frac{\partial \omega}{\partial p}$

where the geostrophic relative vorticity is defined as: $\zeta_g = \frac{1}{f_0} \nabla^2 \phi$

FUNDAMENTALS PV- QG framework

2) Using the thermodynamic equation, let's find an expression for $f_0 \frac{\partial \omega}{\partial p}$:

$$f_0 \frac{\partial \omega}{\partial p} = f_0 \frac{\partial}{\partial p} \left[\frac{1}{\sigma} \frac{\partial}{\partial t} \left(-\frac{\partial \phi}{\partial p} \right) \right] - f_0 \frac{\partial}{\partial p} \left\{ \frac{1}{\sigma} \left[-\vec{V}_g \cdot \vec{\nabla} \left(-\frac{\partial \phi}{\partial p} \right) \right] \right\}$$

$$\cancel{\frac{\partial \vec{V}_g}{\partial p} \cdot \frac{1}{\sigma} \vec{\nabla} \left(-\frac{\partial \phi}{\partial p} \right)} - \vec{V}_g \cdot \frac{\partial}{\partial p} \left[\frac{1}{\sigma} \vec{\nabla} \left(-\frac{\partial \phi}{\partial p} \right) \right]$$

Thermal wind: $\vec{V}_T \equiv -\frac{\partial \vec{V}_g}{\partial p} = \frac{R_d}{f_0 p} \hat{k} \times \vec{\nabla} T$

$\propto T$ (from the hydrostatic relation)

resulting in the expression: $f_0 \frac{\partial \omega}{\partial p} = f_0 \frac{\partial}{\partial p} \left[\frac{1}{\sigma} \frac{\partial}{\partial t} \left(-\frac{\partial \phi}{\partial p} \right) \right] + f_0 \vec{V}_g \cdot \frac{\partial}{\partial p} \left[\frac{1}{\sigma} \vec{\nabla} \left(-\frac{\partial \phi}{\partial p} \right) \right]$

FUNDAMENTALS PV- QG framework

3) And now substitute this expression in the vorticity equation:

$$\frac{\partial \zeta_g}{\partial t} = -\vec{V}_g \cdot \vec{\nabla}(\zeta_g + f) - f_0 \frac{\partial}{\partial p} \left[\frac{1}{\sigma} \frac{\partial}{\partial t} \left(\frac{\partial \phi}{\partial p} \right) \right] - f_0 \vec{V}_g \cdot \frac{\partial}{\partial p} \left[\frac{1}{\sigma} \vec{\nabla} \left(\frac{\partial \phi}{\partial p} \right) \right]$$

$$\sigma \approx \sigma_{ref}(p)$$

$$f = f(y)$$

$$\frac{\partial}{\partial t} \left[\zeta_g + f + \frac{\partial}{\partial p} \left(\frac{f_0}{\sigma} \frac{\partial \phi}{\partial p} \right) \right] = -\vec{V}_g \cdot \vec{\nabla} \left[\zeta_g + f + \frac{\partial}{\partial p} \left(\frac{f_0}{\sigma} \frac{\partial \phi}{\partial p} \right) \right]$$

and therefore we can define the quantity:

$$QGPV \equiv \zeta_g + f + \frac{\partial}{\partial p} \left(\frac{f_0}{\sigma} \frac{\partial \phi}{\partial p} \right)$$

such that ...

(Quasigeostrophic potential vorticity)

FUNDAMENTALS PV- QG framework

a) Conservation principle:

$$\frac{D_g}{Dt}(QGPV) = 0$$

In an adiabatic and frictionless atmosphere, it is conserved following the geostrophic motion

b) Invertibility principle:

QGPV field

Function of ϕ

Balance condition

Geostrophic balance
(Requires $Ro \rightarrow 0$)

Boundary conditions

On ϕ / ϕ_p

Linear operator
(anomalies)

A balance flow can be calculated from the *QGPV* field:

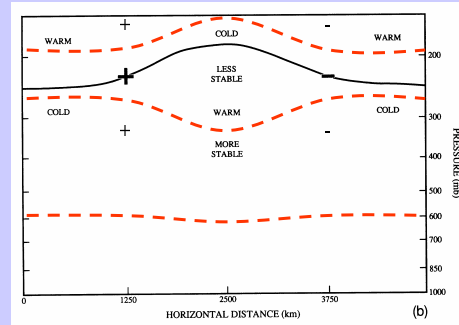
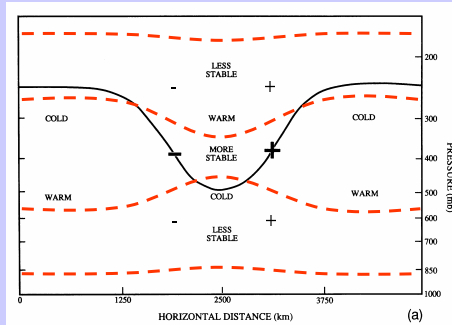
$$\phi, \vec{V}_g, T$$

c) About the anomalies:

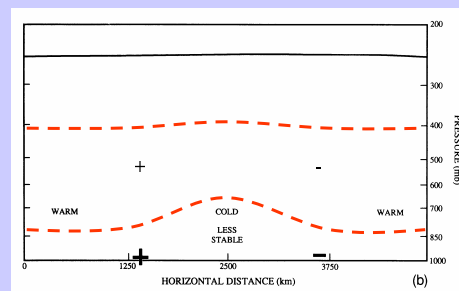
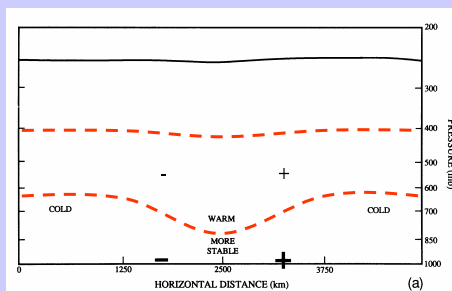
$$QGPV \begin{cases} \zeta_g + f & \text{Coriolis parameter increases with latitude} \\ \frac{\partial}{\partial p} \left(\frac{f_0}{\sigma} \frac{\partial \phi}{\partial p} \right) = -\frac{\partial}{\partial p} \left(\frac{f_0 R_d}{\sigma} T \right) \approx -\frac{f_0 R_d}{\sigma} \frac{\partial T}{\partial p} & \begin{matrix} < 0 \text{ in troposphere} \\ > 0 \text{ in stratosphere} \end{matrix} \end{cases}$$

- *QGPV* is typically higher/lower in high/low latitude, stratospheric/tropospheric air: Source of +/- anomalies
- +/- anomalies are consistent with positive/negative relative vorticity and (or ?) enhanced/reduced stability

FUNDAMENTALS PV - Upper Level PV Anomalies



FUNDAMENTALS PV - Surface Thermal Anomalies



COMPARISON – Ertel's Potential Vorticity

$$EPV \equiv \frac{1}{\rho} \vec{\eta} \cdot \vec{\nabla} \theta$$

a) Conservation principle:

$$\frac{D}{Dt}(EPV) = 0$$

In an adiabatic and frictionless atmosphere, it is conserved **following air-parcel motion** (even if the atmosphere is nonhydrostatic)

b) Invertibility principle:

$$\text{Balance condition} + \text{EPV field} + \text{Boundary conditions}$$

Nonlinear balance
(very small irrot.wind)
(Accurate for $Ro \rightarrow 1$)

Under the same
scale analysis

Nonlinear operator
(anomalies !!!)

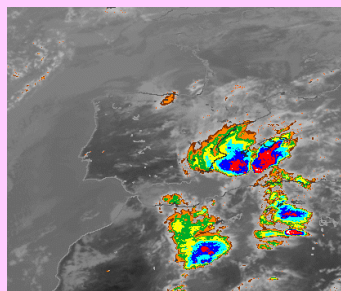
A balance flow can be calculated from the EPV field:

$$\phi, \vec{V}_\psi, T$$

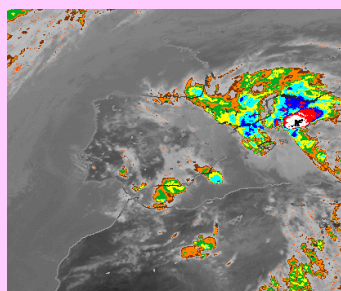
c) About the anomalies:

Same qualitative picture as for the QGPV anomalies

INFRARED METEOSAT



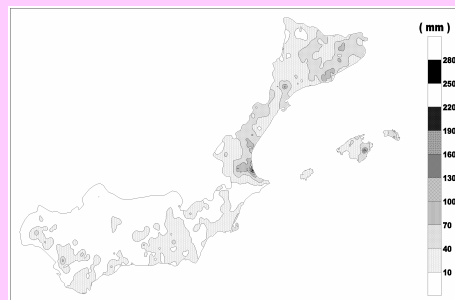
28th / 12 UTC



29th / 12 UTC

THE EVENT (28-29 Sept. 1994)

PRECIPITATION



28th / 07 UTC → 30th / 07 UTC

The cyclone progressed northwards during the episode
Main MCSs developed over the sea (**strong QG forcing ?**)
Heavy precipitation and flash floods in eastern Spain

CONTROL NUMERICAL SIMULATION

* PSU-NCAR mesoscale model (non-hydrostatic version MM5)

* **Simulation:**

- **2 domains:** 82x82x31 (60 and 20 km)
- **Interaction:** two-way
- **I.C and B.C:** NCEP global analysis + Surface and Upper air obs.
- **Period:** 48 h, from 00 UTC 28 September 1994

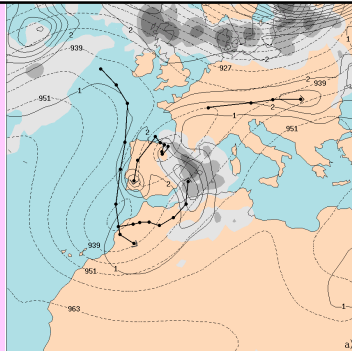
* **Physical parameterizations:**

- **PBL:** Based on Blackadar (1979) scheme (Zhang and Anthes 1982)
- **Ground temperature:** Force-restore slab model (Blackadar 1979)
- **Radiation fluxes:** Considering cloud cover (Benjamin 1983)
- **Resolved-scale microphysics:**
Cloud water, rainwater, cloud ice and snow (Dudhia 1989)
- **Parameterized convection:**
60 km: Betts-Miller (1986)
20 km: Kain-Fritsh (1990)

SYNOPTIC ASPECTS

Two rotating upper-level
positive PV anomalies

Strong low-tropospheric
warm advection

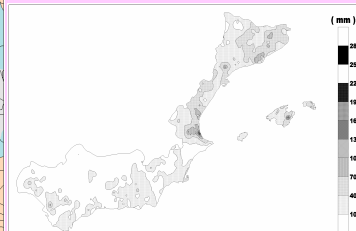
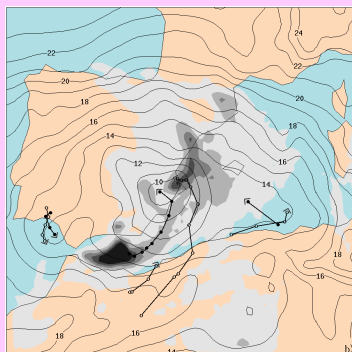


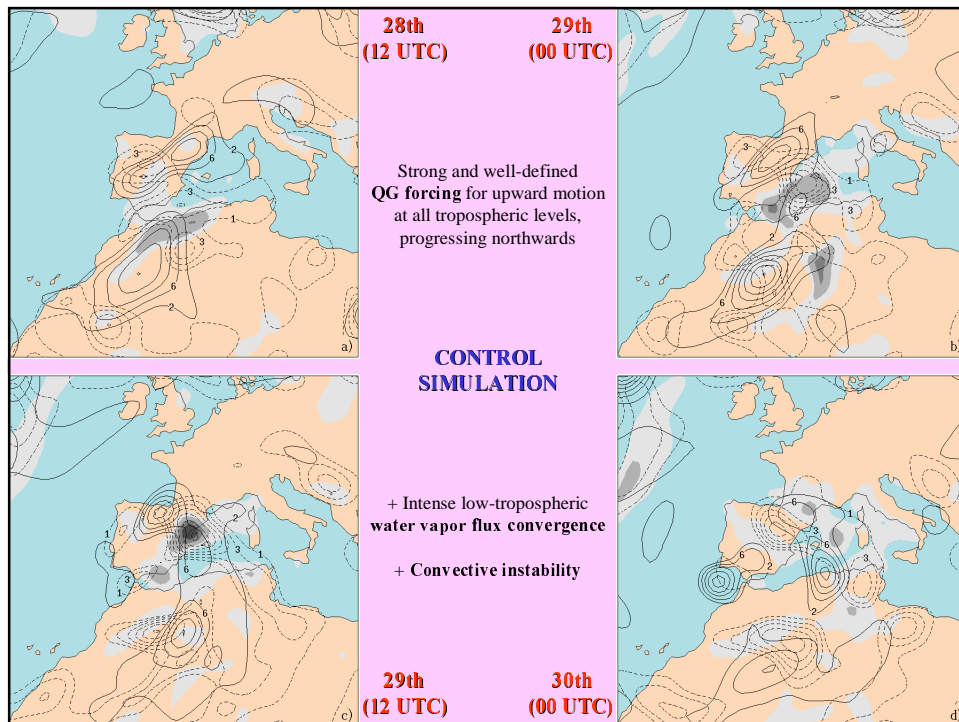
CONTROL SIMULATION

MESOSCALE FORECAST

Intense, broad and
mobile surface cyclone

Heavy precipitation in
agreement with observations





SENSITIVITY TO THE UPPER LEVEL PV ANOMALIES (motivation)

* The two embedded **upper-level PV centres** seem to be playing an **important role** for the evolution, intensity and spatial extent of the **surface cyclone**

* How a potential analysis and/or forecast **error** in the representation of these **PV anomalies** might affect the **mesoscale forecast** ?



* **Sensitivity analysis** based on additional simulations with perturbed initial conditions

* A **balanced flow** associated with each anomaly **must be found** that can be used to alter the model initial conditions in a physically consistent way without introducing any significant noise in the model → **Piecewise PV inversion**

PIECEWISE PV INVERSION TECHNIQUE (Davis and Emanuel; MWR 1991)

1) Balanced flow (ϕ , ψ) given instantaneous distribution of Ertel's PV (q):

* Charney (1955) nonlinear balance equation

$$\nabla^2 \phi = \nabla \cdot f \nabla \psi + 2m^2 \left[\frac{\partial^2 \psi}{\partial x^2} \frac{\partial^2 \psi}{\partial y^2} - \left(\frac{\partial^2 \psi}{\partial x \partial y} \right)^2 \right]$$

f Coriolis parameter m map-scale factor

* Approximate form of Ertel's PV

$$q = \frac{g\kappa\pi}{p} \left[(f + m^2 \nabla^2 \psi) \frac{\partial^2 \phi}{\partial \pi^2} - m^2 \left(\frac{\partial^2 \psi}{\partial x \partial \pi} \frac{\partial^2 \phi}{\partial x \partial \pi} + \frac{\partial^2 \psi}{\partial y \partial \pi} \frac{\partial^2 \phi}{\partial y \partial \pi} \right) \right]$$

p pressure g gravity $\kappa = Rd/Cp$ $\pi = Cp(p/p_0)^\kappa$

* **Bounday conditions** Lateral (Dirichlet) / Top and Bottom (Neumann): $\partial \phi / \partial \pi = f \partial \psi / \partial \pi = -\theta$
 θ potential temperature

2) Reference state: Balanced flow ($\bar{\phi}$, $\bar{\psi}$) given time mean distribution of Ertel's PV (\bar{q}):

* Same equations as in **1)**, except using time mean fields instead of instantaneous fields

3) Perturbation fields (ϕ' , ψ' , q') given by the definitions: $(q, \phi, \psi) = (\bar{q}, \bar{\phi}, \bar{\psi}) + (q', \phi', \psi')$

PIECEWISE PV INVERSION TECHNIQUE

4) We consider that q' is partitioned into N portions or anomalies: $q' = \sum_{n=1}^N q_n$

5) Piecewise inversion: (ϕ_n , ψ_n) associated with q_n ?

... and requiring:

$$\phi' = \sum_{n=1}^N \phi_n$$

$$\psi' = \sum_{n=1}^N \psi_n$$

...After substitution of the above summations in the balance and PV equations and some rearrangements of the nonlinear terms:

$$\nabla^2 \phi_n = \nabla \cdot f \nabla \psi_n + 2m^2 \left(\frac{\partial^2 \psi^*}{\partial x^2} \frac{\partial^2 \psi_n}{\partial y^2} + \frac{\partial^2 \psi^*}{\partial y^2} \frac{\partial^2 \psi_n}{\partial x^2} - 2 \frac{\partial^2 \psi^*}{\partial x \partial y} \frac{\partial^2 \psi_n}{\partial y \partial x} \right)$$

$$q_n = \frac{g\kappa\pi}{p} \left[(f + m^2 \nabla^2 \psi^*) \frac{\partial^2 \phi_n}{\partial \pi^2} + m^2 \frac{\partial^2 \phi^*}{\partial \pi^2} \nabla^2 \psi_n - m^2 \left(\frac{\partial^2 \phi^*}{\partial x \partial \pi} \frac{\partial^2 \psi_n}{\partial x \partial \pi} + \frac{\partial^2 \phi^*}{\partial y \partial \pi} \frac{\partial^2 \psi_n}{\partial y \partial \pi} \right) - m^2 \left(\frac{\partial^2 \psi^*}{\partial x \partial \pi} \frac{\partial^2 \phi_n}{\partial x \partial \pi} + \frac{\partial^2 \psi^*}{\partial y \partial \pi} \frac{\partial^2 \phi_n}{\partial y \partial \pi} \right) \right]$$

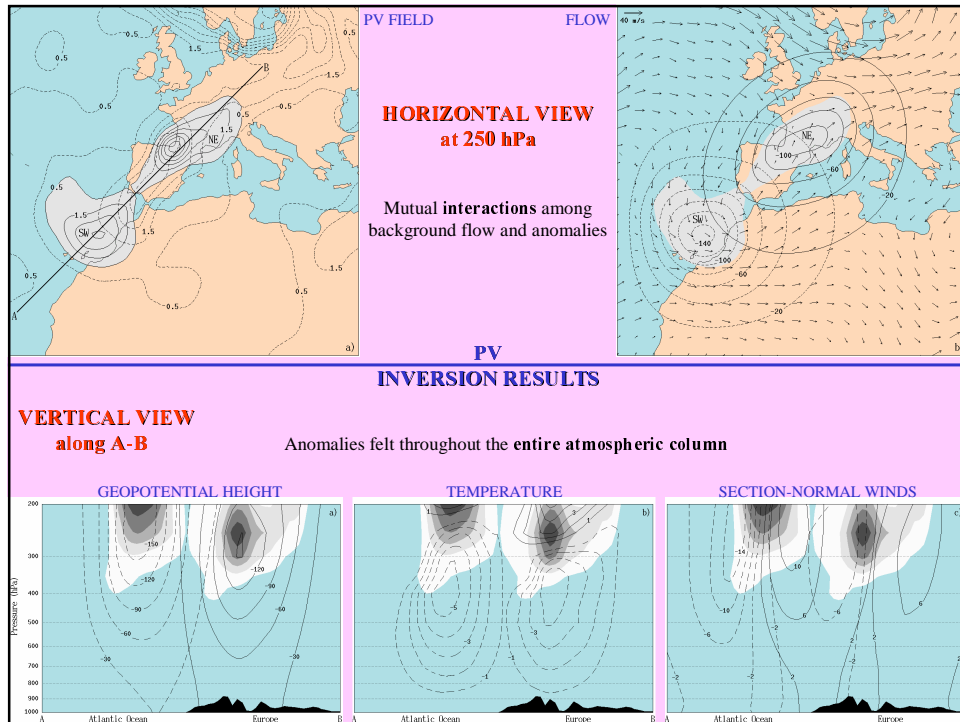
where $(\)^* = \bar{(\)} + \frac{1}{2}(\)'$

Boundary conditions: Lateral (homogeneous) / Top and bottom (using θ_r)

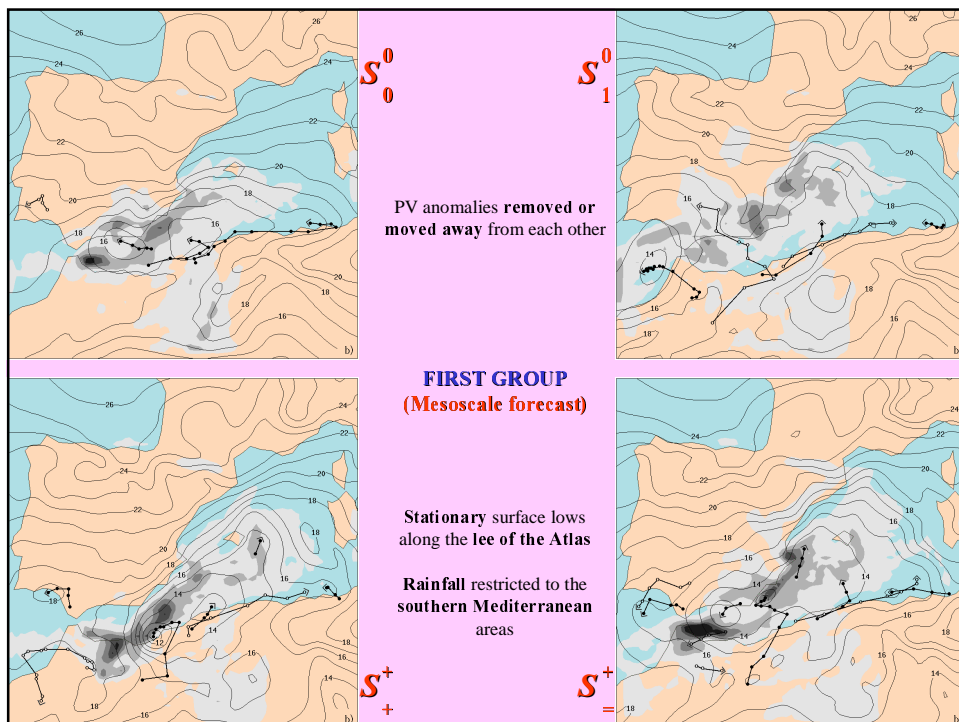
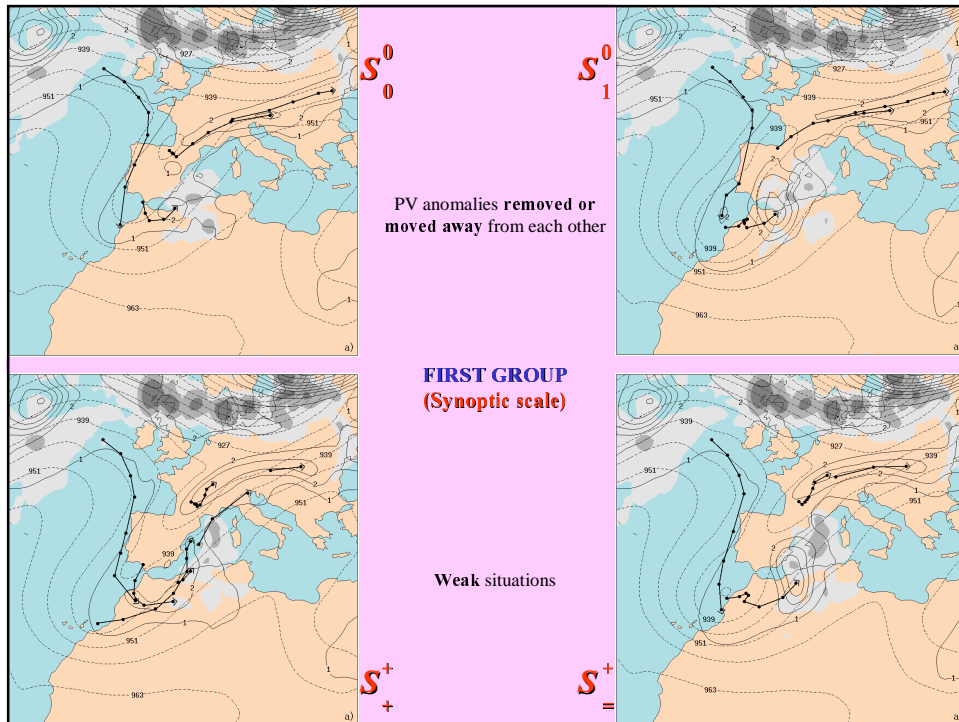
At 00 UTC 28 September 1994, using the NCEP-based isobaric analysis

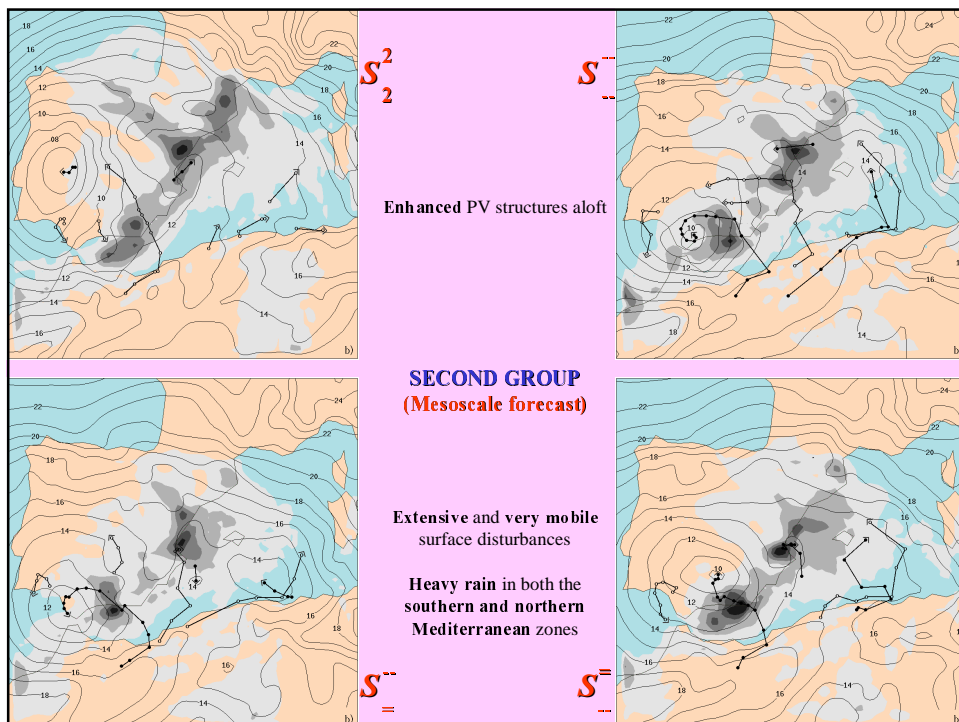
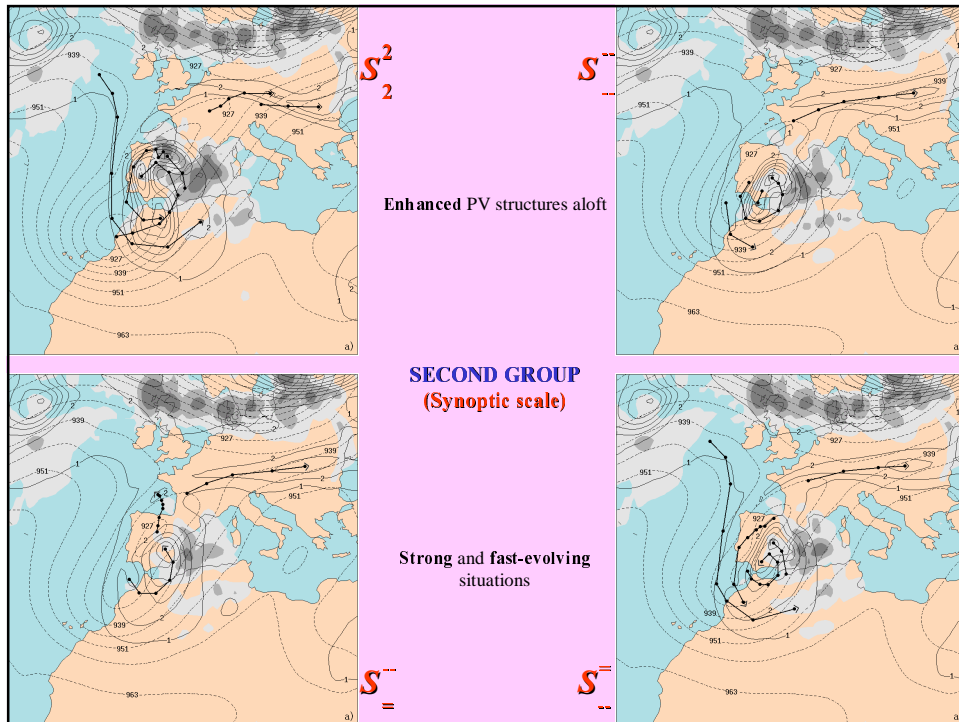
* **In our case study:** **Reference state:** 6-day time average about 00 UTC 28 September

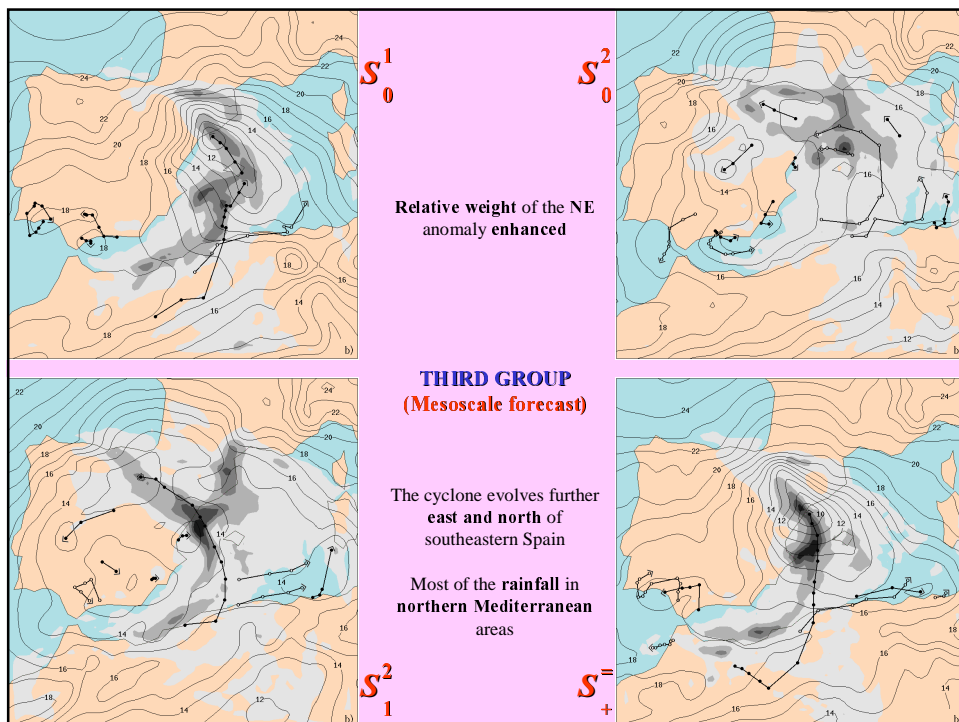
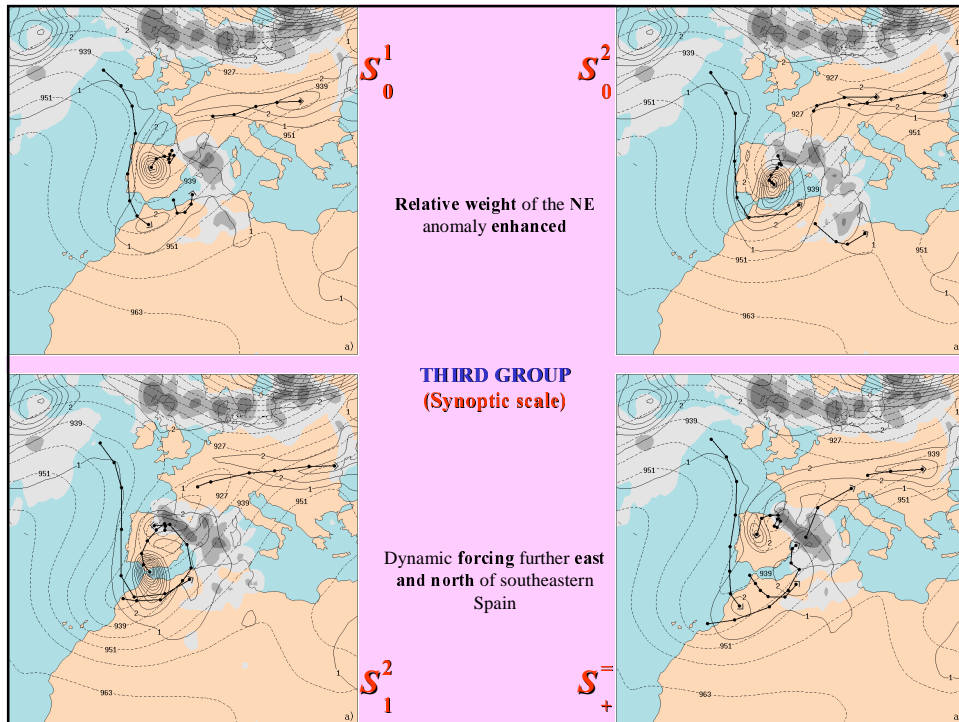
Anomalies: positive PV perturbations above 500 hPa **SW** and **NE** of Gulf of Cádiz

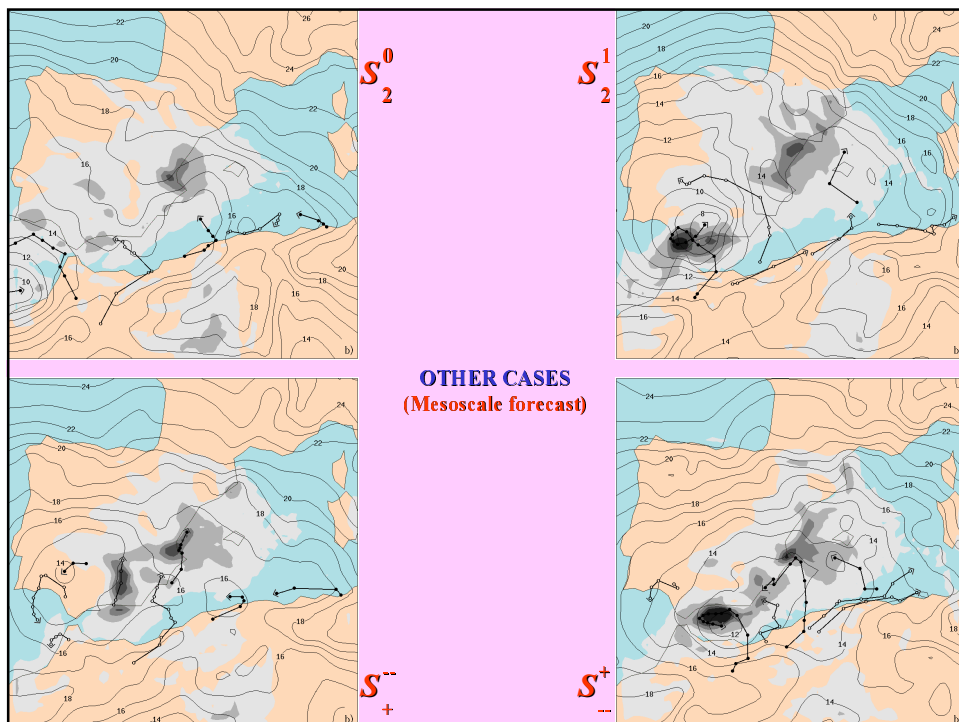
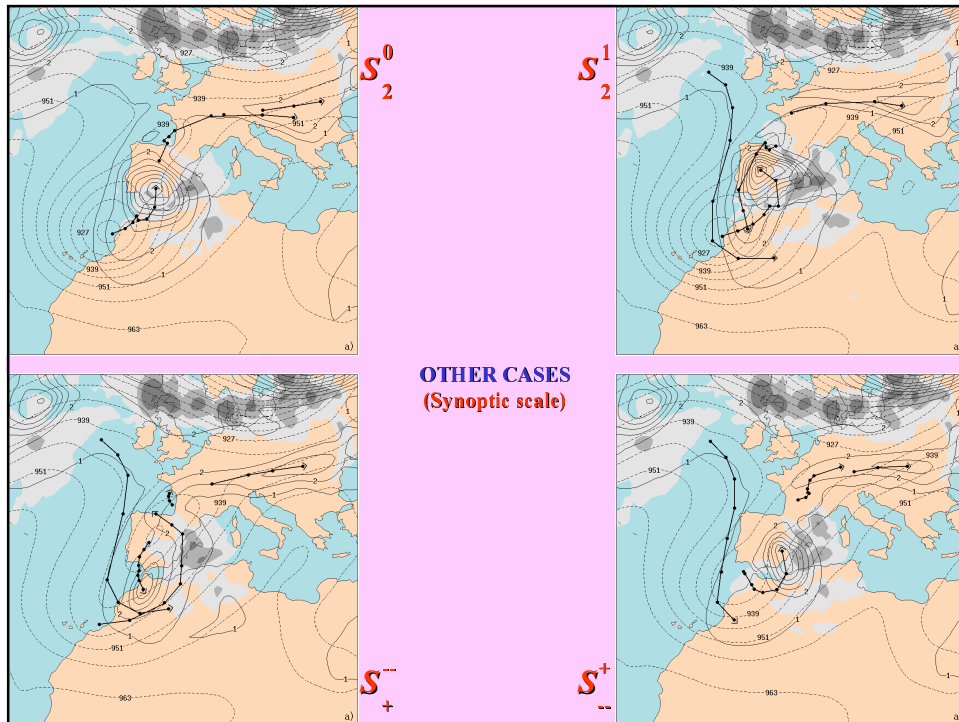


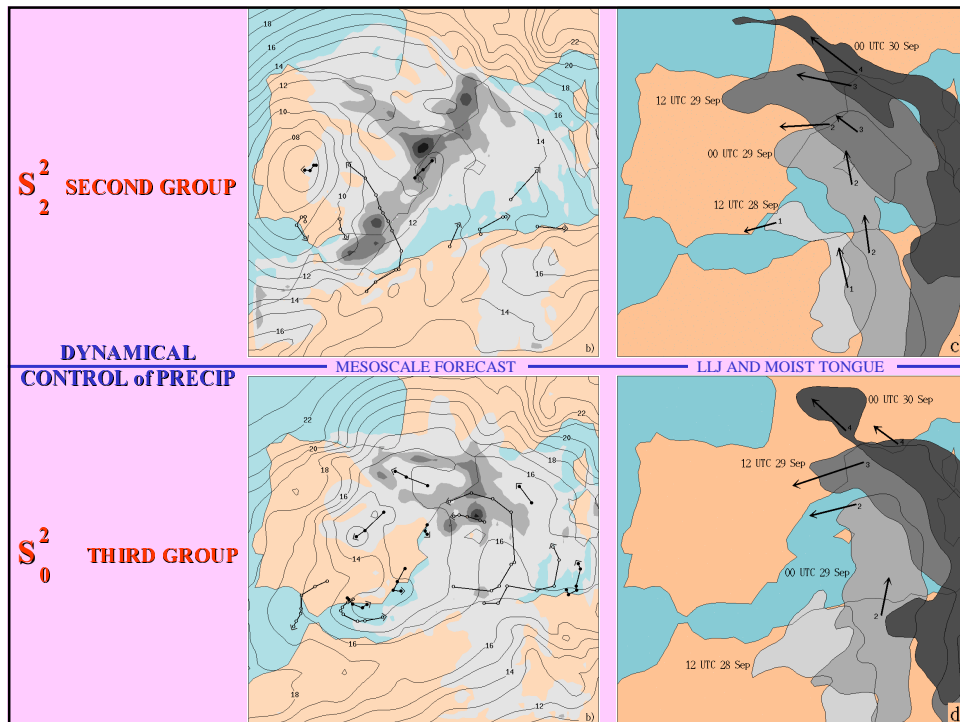
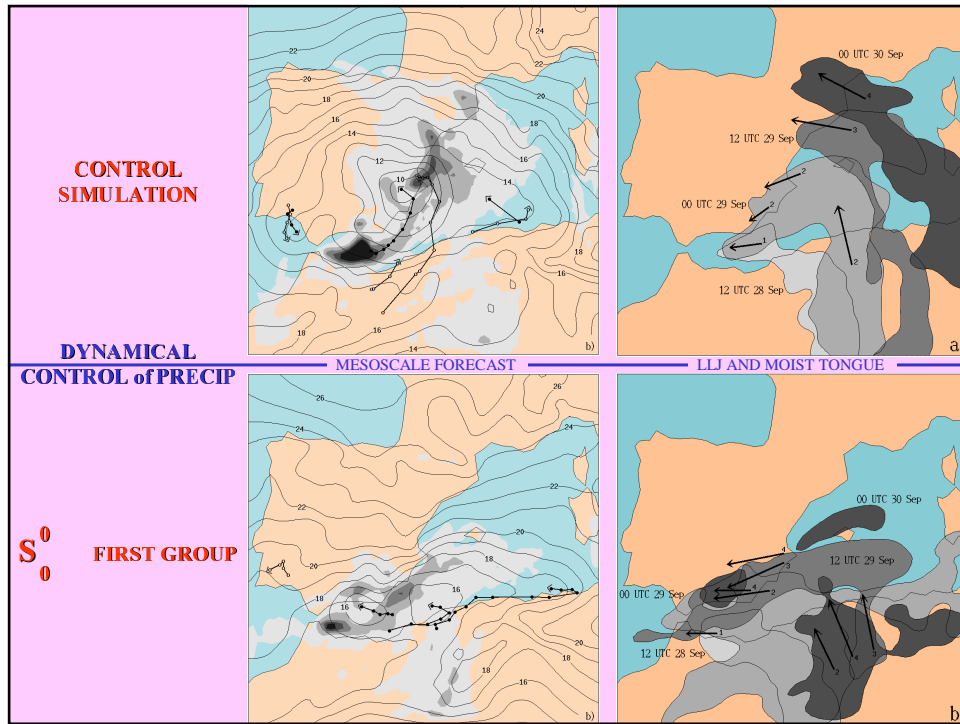
| SENSITIVITY EXPERIMENTS | | | |
|---|----------------|---|--|
| By adding and/or subtracting the PV-inverted balanced fields (geopotential, temperature and wind) into the model initial conditions | | | |
| Sensitivity to the intensity | | Sensitivity to the position | |
| (One or both PV anomalies removed or doubled) | | (One or both PV anomalies shifted 425 km along A-B) | |
| Experiment | SW anomaly | NE anomaly | |
| S_0^0 | Removed | Removed | |
| S_2^2 | Doubled | Doubled | |
| S_1^0 | Unchanged | Removed | |
| S_2^0 | Doubled | Removed | |
| S_1^1 | Removed | Unchanged | |
| S_0^2 | Removed | Doubled | |
| S_2^1 | Doubled | Unchanged | |
| S_1^2 | Unchanged | Doubled | |
| Experiment | SW anomaly | NE anomaly | |
| S_-^- | Moved inwards | Moved inwards | |
| S_+^+ | Moved outwards | Moved outwards | |
| $S_-^=$ | Unchanged | Moved inwards | |
| S_+^- | Moved outwards | Moved inwards | |
| $S_-^=$ | Moved inwards | Unchanged | |
| S_+^+ | Moved inwards | Moved outwards | |
| S_-^+ | Moved outwards | Unchanged | |
| $S_+^=$ | Unchanged | Moved outwards | |

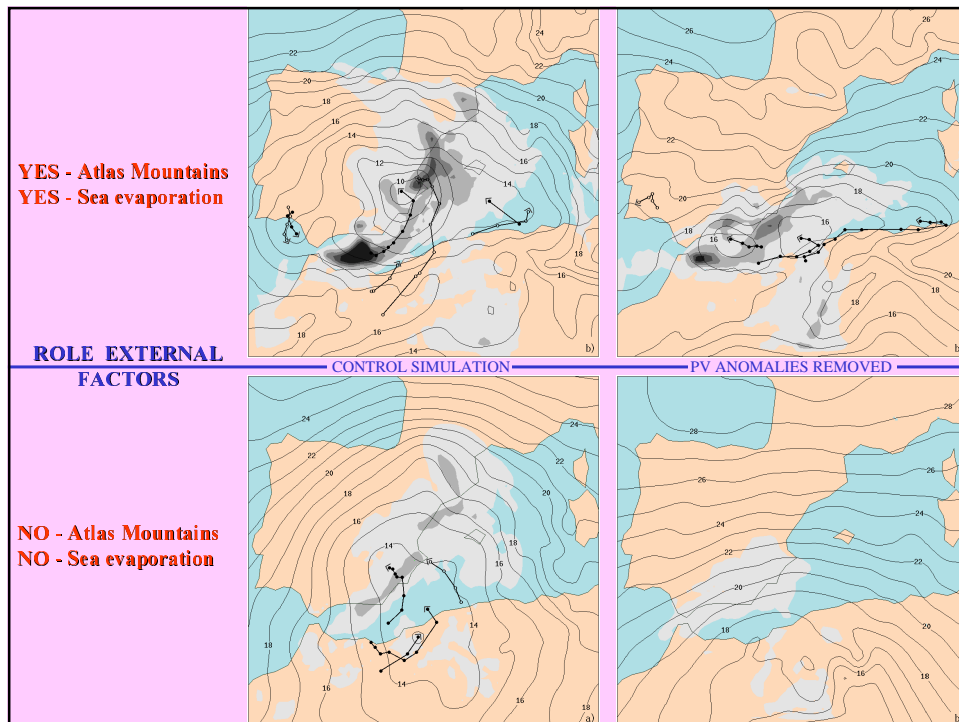










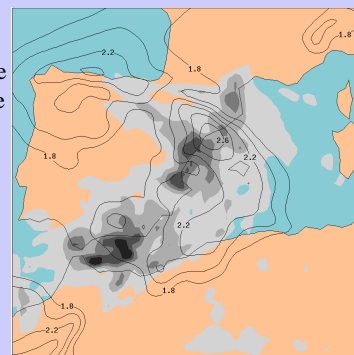


CONCLUSIONS

* Track, shape and intensity of the **surface cyclone** and the corresponding **rainfall pattern** are very **sensitive** to the embedded upper-level **PV anomalies**



A potential **error** in the initial representation of the **anomalies** can be **critical**



SLP-TENDENCY and RAINFALL (STD)

* The **external factors** induced an **appreciable** modulation of the surface circulation and enhanced the efficiency of the system as a rainfall producer, **but** the cyclogenesis over the southern Mediterranean and its progression to the north must be attributed **mostly** to the action of the upper-level **PV anomalies**

* The combined application of **piecewise PV inversion + numerical simulation** offers a **valuable framework** from which the effects of **dynamical features** of the flow can be studied

INTRODUCTION- Lecture 3

LIFE CYCLE OF AN INTENSE MEDITERRANEAN CYCLONE

PV THINKING → An analysis of the cyclone event in terms of the **impacts** and **interactions** of dry and moist **PV anomalies** (and mean flow)

Beyond a qualitative analysis, **how** can these impacts and interactions be **quantified ???**



PV-BASED PROGNOSTIC SYSTEM + FACTOR SEPARATION