

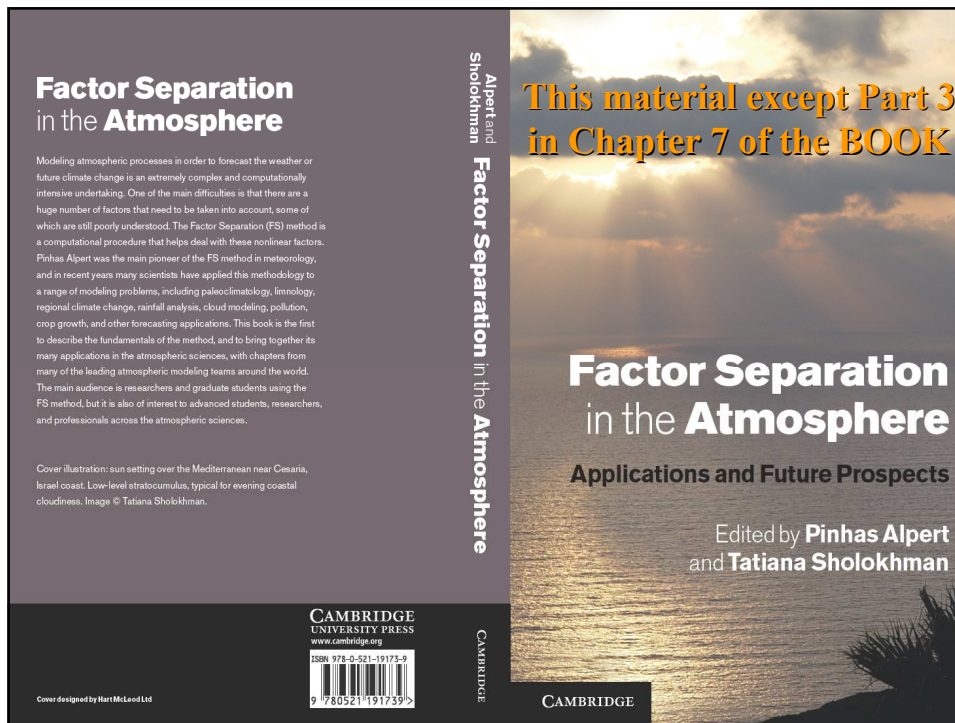
# APPLICATION OF FACTOR SEPARATION AND PV INVERSION TO HEAVY RAINFALL AND CYCLOGENESIS EVENTS: MEDITERRANEAN EXAMPLES

*Romualdo Romero March*



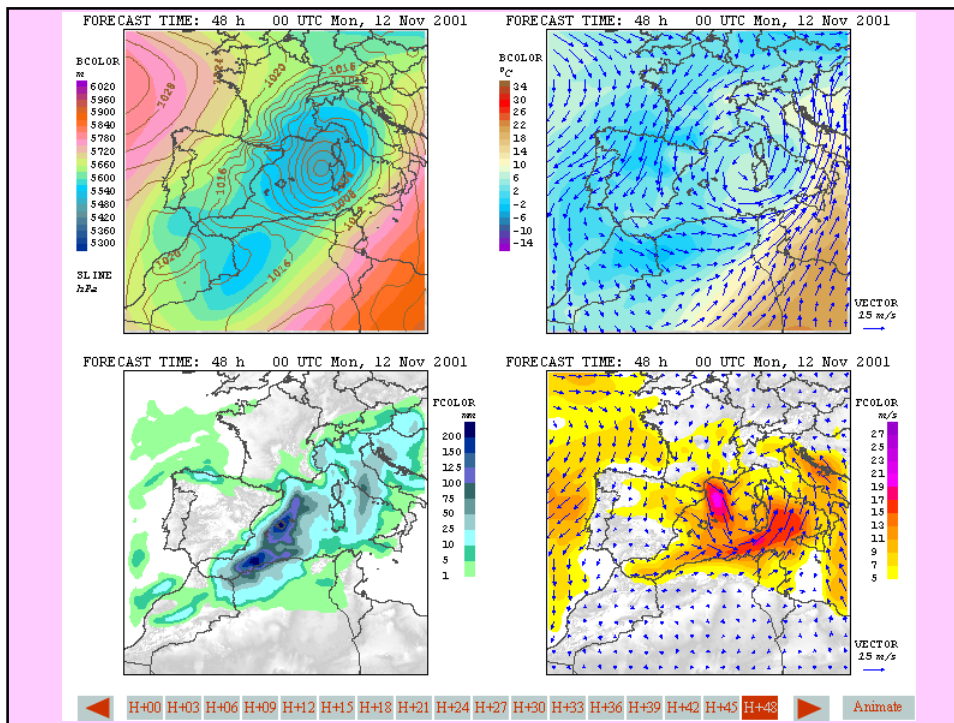
## STRUCTURE

- **Introduction:** The *Factor Separation* technique
- **Part 1:** *Boundary* factors and *Model Physics* factors  
Romero, R., Doswell, C. A. III and Ramis, C., 2000: Mesoscale numerical study of two cases of long-lived quasistationary convective systems over eastern Spain. *Mon. Wea. Rev.*, 128, 3731-3751.
- **Part 2:** *Dynamical* factors - Piecewise *PV inversion*  
Romero, R., 2001: Sensitivity of a heavy rain producing Western Mediterranean cyclone to embedded potential vorticity anomalies. *Quart. J. R. Meteorol. Soc.*, 127, 2559-2597.
- **Part 3:** *Ensemble* Prediction - *PV-error* climatology  
Vich, M., R. Romero, and H. E. Brooks/V. Homar, 2011: Ensemble prediction of Mediterranean high-impact events using potential vorticity perturbations. *Atmos. Res.*, 102, 224-241 (Part I) and 311-319 (Part II).
- **Part 4:** Quantitative implementation of *PV-thinking*  
Romero, R., 2008: A method for quantifying the impacts and interactions of potential-vorticity anomalies in extratropical cyclones. *Quart. J. R. Meteorol. Soc.*, 134, 385-402.

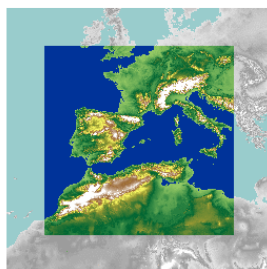


## THE STUDY OF ATMOSPHERIC PHENOMENA

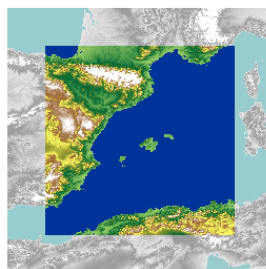
- Observations (limited in number, space and time)
- Theory (requires simplifications)
- Experimentation (*Numerical Modeling*)



- *Multiscale* perspective of the problem



DOMAIN 1 (22.5 km resolution)

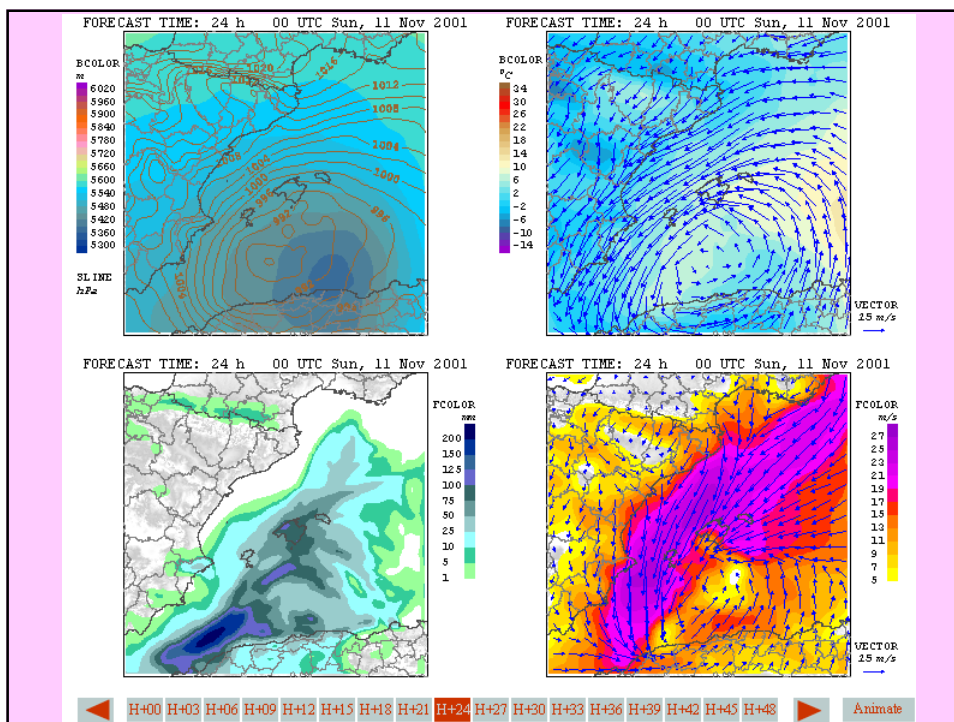
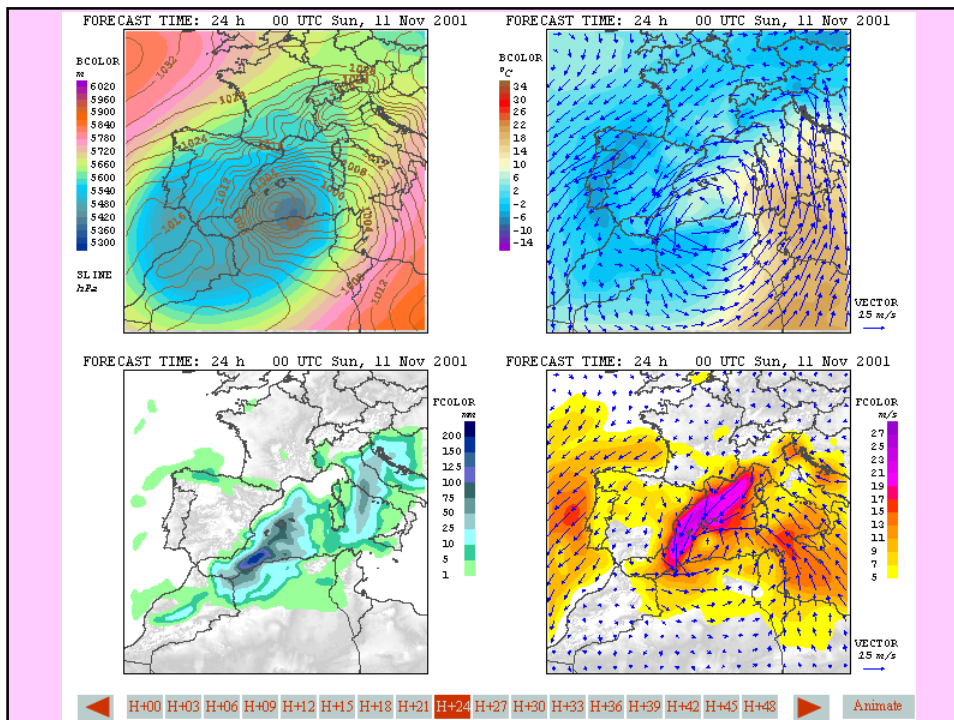


DOMAIN 2 (7.5 km resolution)

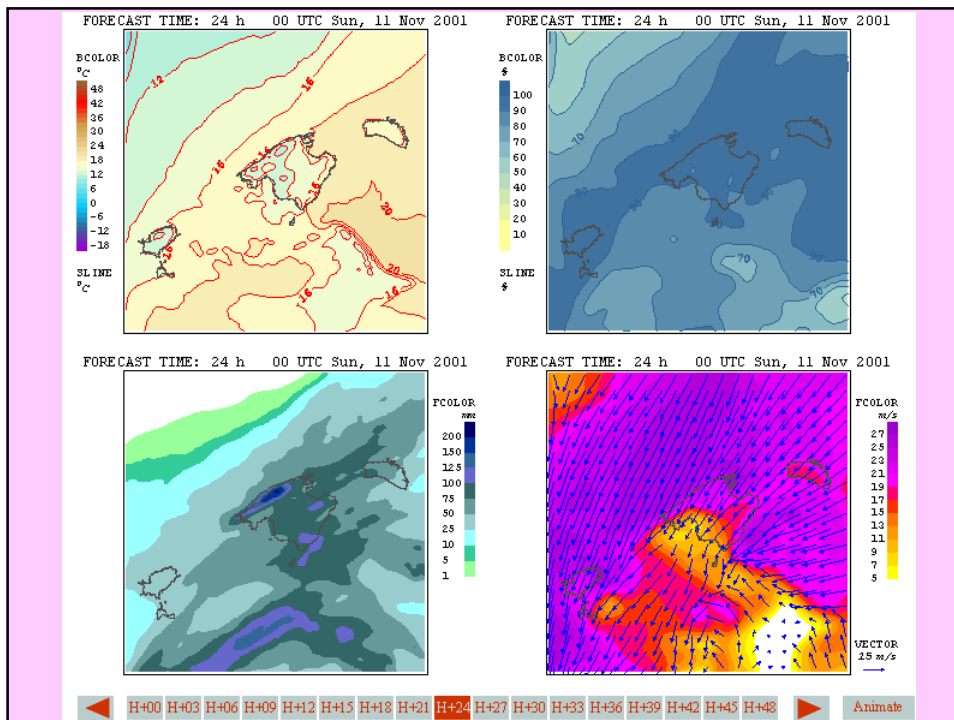


DOMAIN 3 (2.5 km resolution)

- Realistic *physical processes* parameterized







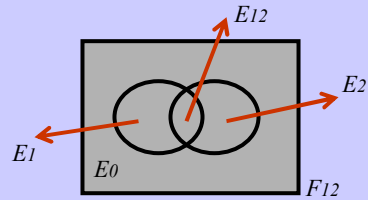
## UNIQUE FEATURE OF NUMERICAL MODELS

- Reasonably *good* control simulation of your case study
- Specifically *designed* simulations (by perturbing factors) (sensitivity studies / factor separation)
- Improved physical *understanding* of your case study

## FACTOR SEPARATION (Stein and Alpert, JAS 1993)

### 2 FACTORS

Run	Factor 1	Factor 2	
$F_{12}$	on	on	$= E_0 + E_1 + E_2 + E_{12}$
$F_1$	on	off	$= E_0 + E_1$
$F_2$	off	on	$= E_0 + E_2$
$F_0$	off	off	$= E_0$



**Unrelated** with factors 1 and 2

$$E_0 = F_0$$

Induced by the **factor 1** (independent of 2)

$$E_1 = F_1 - F_0$$

Induced by the **factor 2** (independent of 1)

$$E_2 = F_2 - F_0$$

Induced by the **synergism** of factors 1 and 2

$$E_{12} = F_{12} - (F_1 + F_2) + F_0$$

\* Generalization:

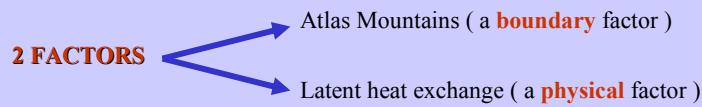
**n FACTORS**  $\longrightarrow$  **2<sup>n</sup> SIMULATIONS**

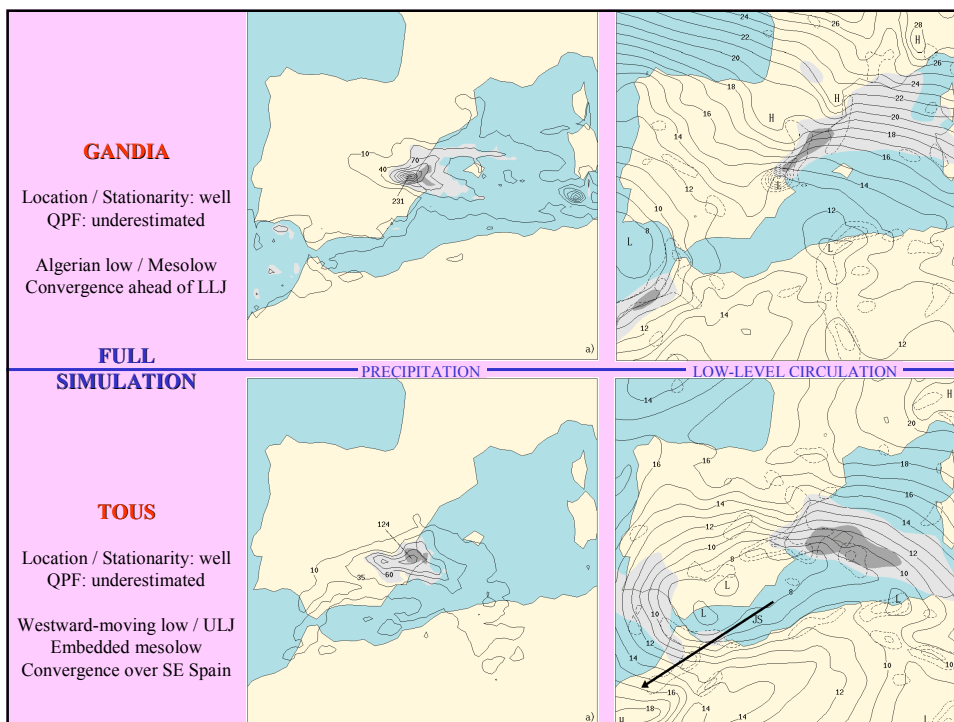
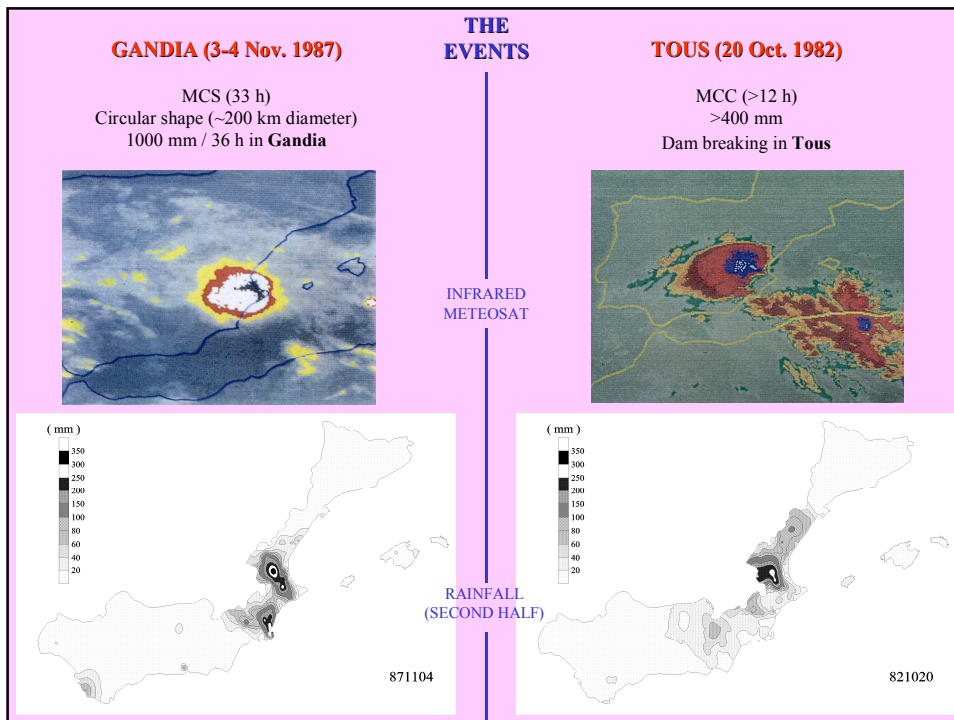
$$E_{i_1 i_2 i_3 \dots i_k} = \sum_{m=0}^k (-1)^{k-m} \left( \sum_{\text{sort}} F_{j_1 j_2 j_3 \dots j_m} \right) \quad 0 \leq k \leq n$$

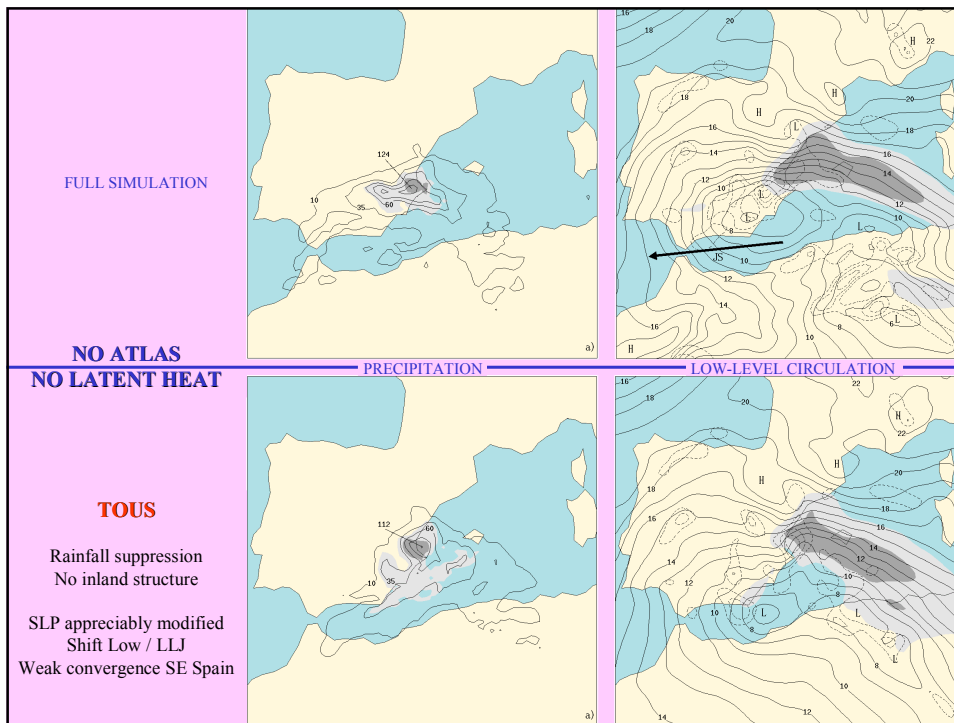
where  $\sum_{\text{sort}}$  is over all groups of  $m$  sorted indices  $j_1 j_2 j_3 \dots j_m$  chosen from  $k$  indices  $i_1 i_2 i_3 \dots i_k$

## PART 1.- CASE STUDIES

### 2 FLASH FLOOD EVENTS OVER EASTERN SPAIN







### FACTOR SEPARATION STUDY

Method of Stein and Alpert (1993)

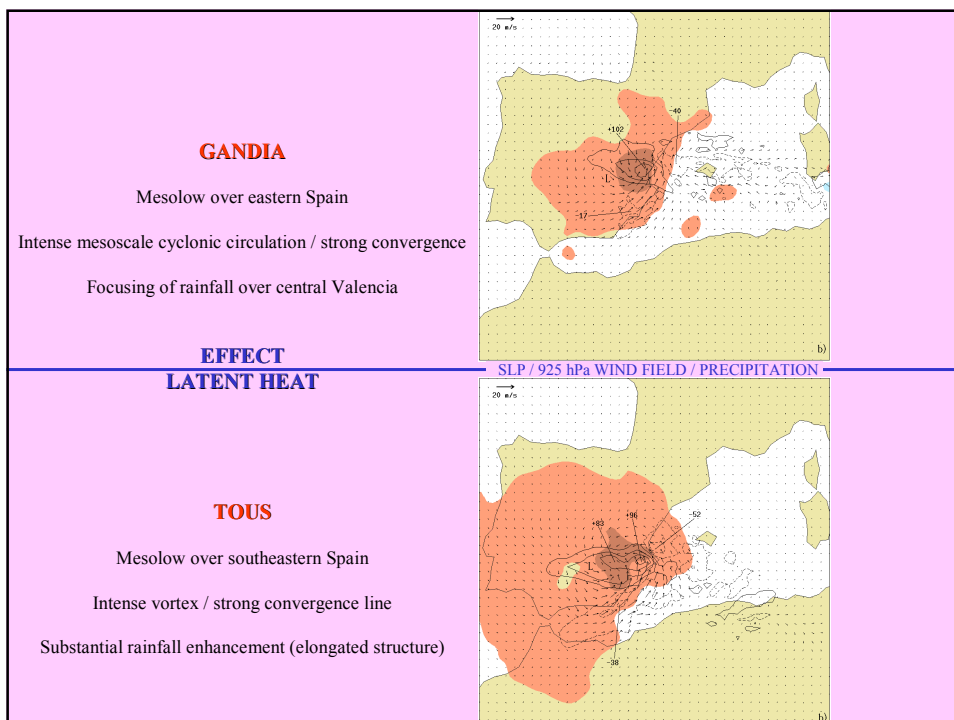
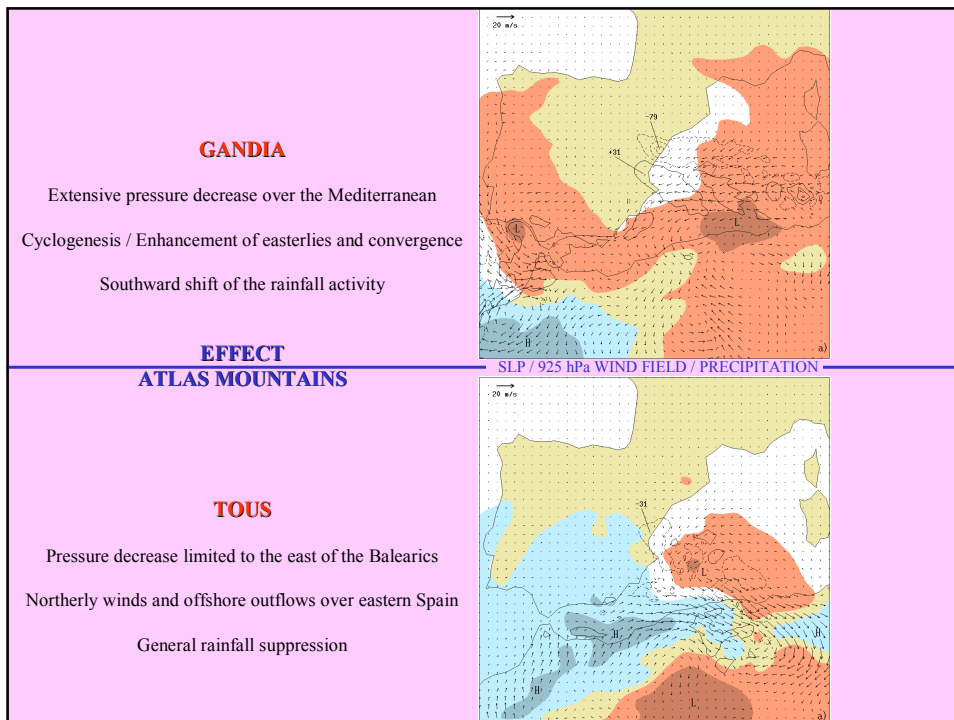
n factors  $\longrightarrow$   $2^n$  simulations

Experiment	Atlas orography	Latent heat exchange
F <sub>0</sub>	no	no
F <sub>1</sub>	yes	no
F <sub>2</sub>	no	yes
F <sub>12</sub>	yes	yes

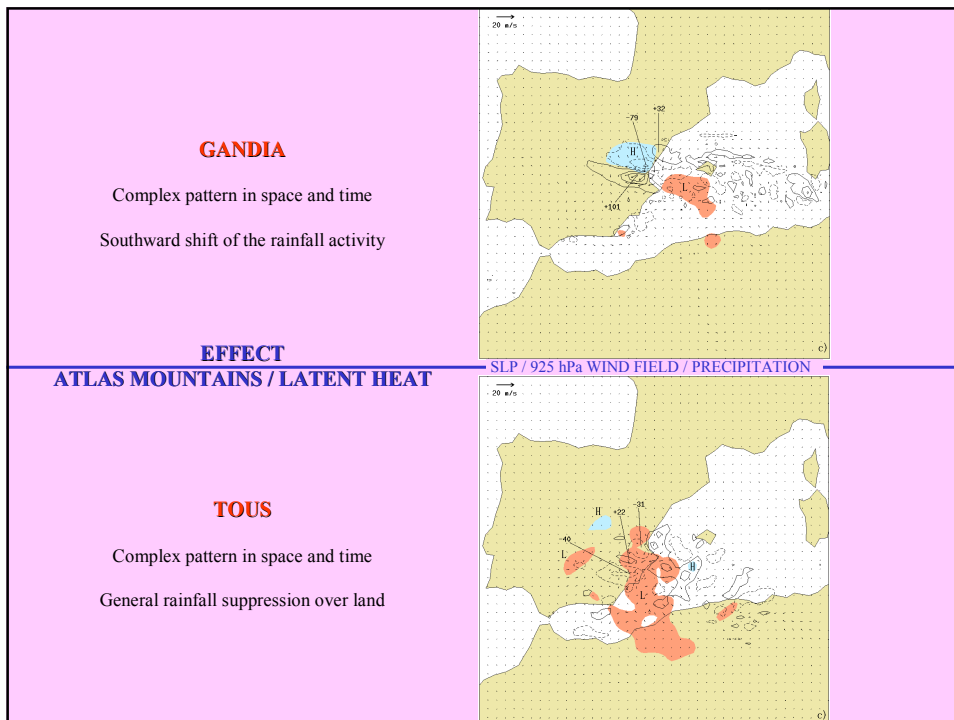
a. Effect of the Atlas Mountains =  $F_1 - F_0$

b. Effect of the Latent heat =  $F_2 - F_0$

c. Effect of the interaction Atlas/Latent heat =  $F_{12} - (F_1 + F_2) + F_0$







### CONCLUSIONS (I) - PART 1

The **numerical modeling** of atmospheric circulations is the most powerful tool available to scientists to develop a better **physical understanding** of the responsible mechanisms and its relation to the **weather or the environment**



#### FACTOR SEPARATION

By **switching on / off** some given factors in the numerical simulations, the **role** played by these factors on our meteorological or environmental problem can be **isolated** !!!

## CONCLUSIONS (II) - PART 1

### 1) Factor separation technique (PROS):

- Numerical simulations can be utilized to obtain the **pure contribution** of any factor to any predicted field, as well as the contributions due to the mutual **interactions** among two or more factors.
- **Easy to apply** (algebraic combinations of model outputs).

### 2) Factor separation technique (CONS):

- **n factors**  $\longrightarrow$   **$2^n$  simulations**  
(e.g. 10 factors would require 1024 simulations, **but** only 56 simulations would be needed to obtain double interactions only).
- The interactions can be **complex** and difficult to interpret

### 3) What about the nature of the factors ?

- **Boundary** and **physical** factors, no problem !
- **But** ... how to deal with **dynamical** factors (**I.C**) ?

## INTRODUCTION - PART 2

### HEAVY RAIN PRODUCING WESTERN MEDITERRANEAN CYCLONE

**FACTORS**  $\longrightarrow$  Two embedded upper level disturbances ( positive PV anomalies )  
( **dynamical** factors )

**How** can the internal features of the flow dynamics (jet streaks, troughs, fronts, etc...) present in the initial conditions be **switched on / off** without compromising the delicate 3-D dynamical balances that govern both the model and actual meteorological fields ???



**PIECEWISE PV INVERSION**

## FUNDAMENTALS PV - QG framework

**a) Conservation principle:**  $\frac{D_g}{Dt}(QG_{PV}) = 0$  In an adiabatic and frictionless atmosphere, it is conserved **following the geostrophic motion**

**b) Invertibility principle:**  $QG_{PV} \text{ field}$  +  $\text{Balance condition}$  +  $\text{Boundary conditions}$

Function of  $\phi$       Geostrophic balance (Requires  $Ro \rightarrow 0$ )      On  $\phi / \phi_p$

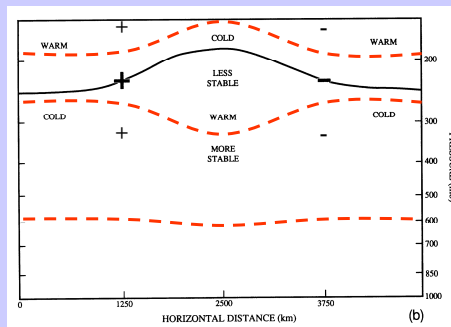
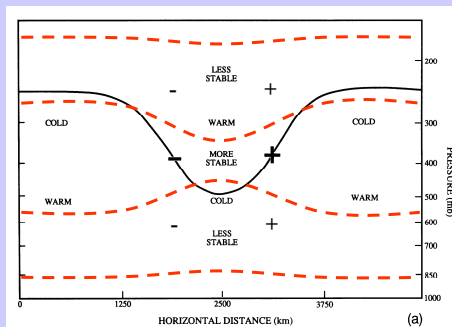
**Linear operator (anomalies)** ↓

A balance flow can be calculated from the  $QG_{PV}$  field:  $\phi, \vec{V}_g, T$

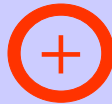
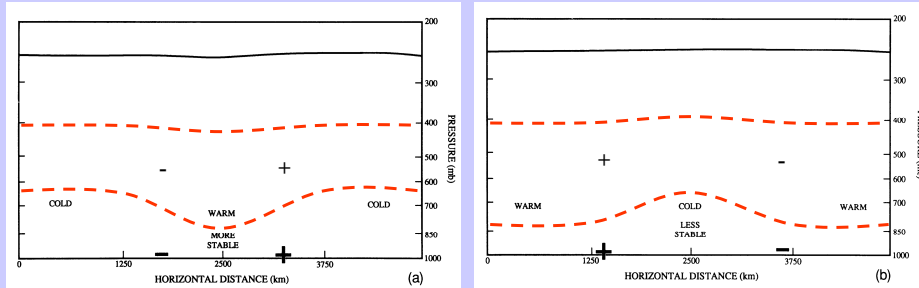
**c) About the anomalies:**  $QG_{PV} \begin{cases} \zeta_g + f & \text{Coriolis parameter increases with latitude} \\ \frac{\partial}{\partial p} \left( \frac{f_0}{\sigma} \frac{\partial \phi}{\partial p} \right) = -\frac{\partial}{\partial p} \left( \frac{f_0 R_d}{\sigma p} T \right) \approx -\frac{f_0 R_d}{\sigma p} \frac{\partial T}{\partial p} & \begin{matrix} < 0 \text{ in troposphere} \\ > 0 \text{ in stratosphere} \end{matrix} \end{cases}$

- $QG_{PV}$  is typically higher/lower in high/low latitude, stratospheric/tropospheric air: Source of +/- anomalies
- +/- anomalies are consistent with positive/negative relative vorticity **and** enhanced/reduced stability

## FUNDAMENTALS PV - Upper Level PV Anomalies



## FUNDAMENTALS PV - Surface Thermal Anomalies



## COMPARISON – Ertel's Potential Vorticity

$$EPV \equiv \frac{1}{\rho} \vec{\eta} \cdot \vec{\nabla} \theta$$

a) Conservation principle:

$$\frac{D}{Dt}(EPV) = 0$$

In an adiabatic and frictionless atmosphere, it is conserved **following air-parcel motion** (even if the atmosphere is nonhydrostatic)

b) Invertibility principle:

$$\text{Balance condition} + \text{EPV field} + \text{Boundary conditions}$$

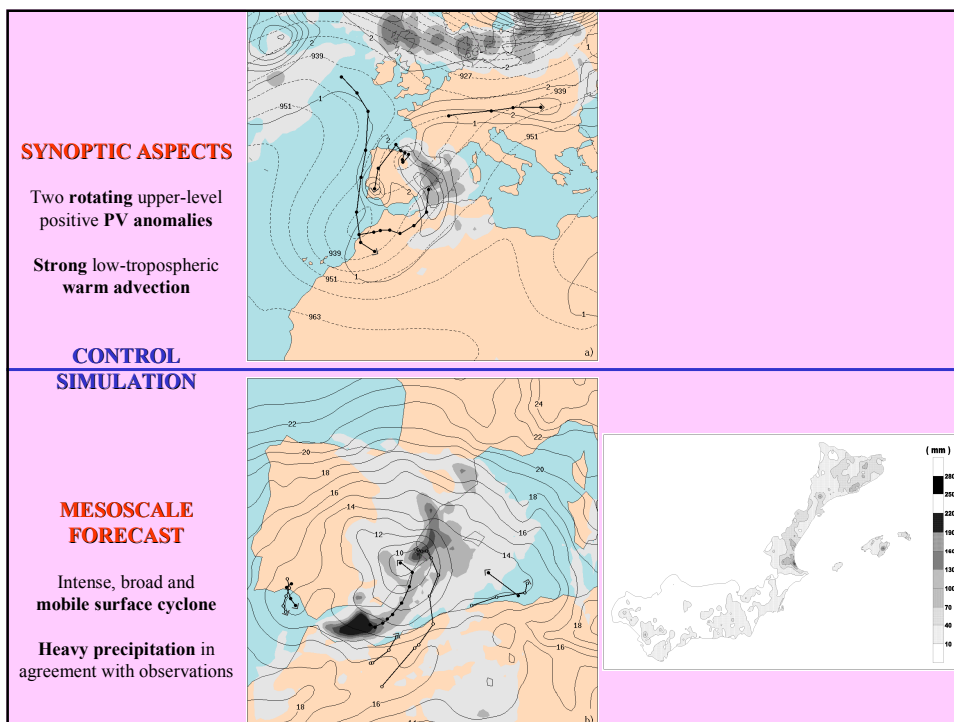
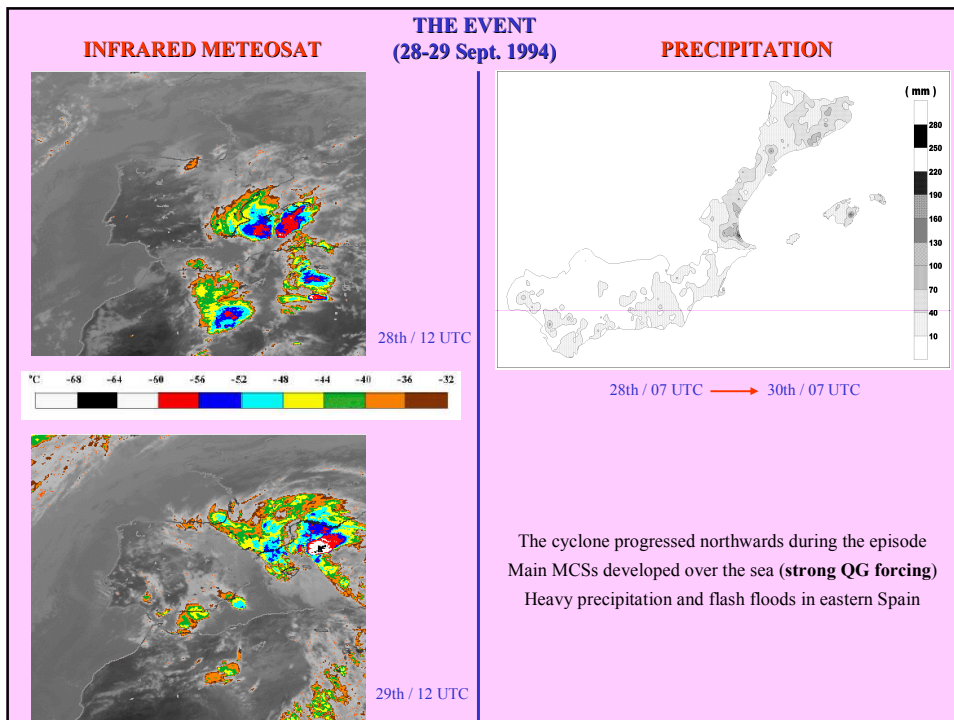
Charney nonlinear balance (very small irrot wind) (Accurate for  $Ro \rightarrow 1$ )      Under the same scale analysis

↓  
Nonlinear operator (anomalies !!!)

A balance flow can be calculated from the EPV field:  $\phi, \vec{V}_\psi, T$

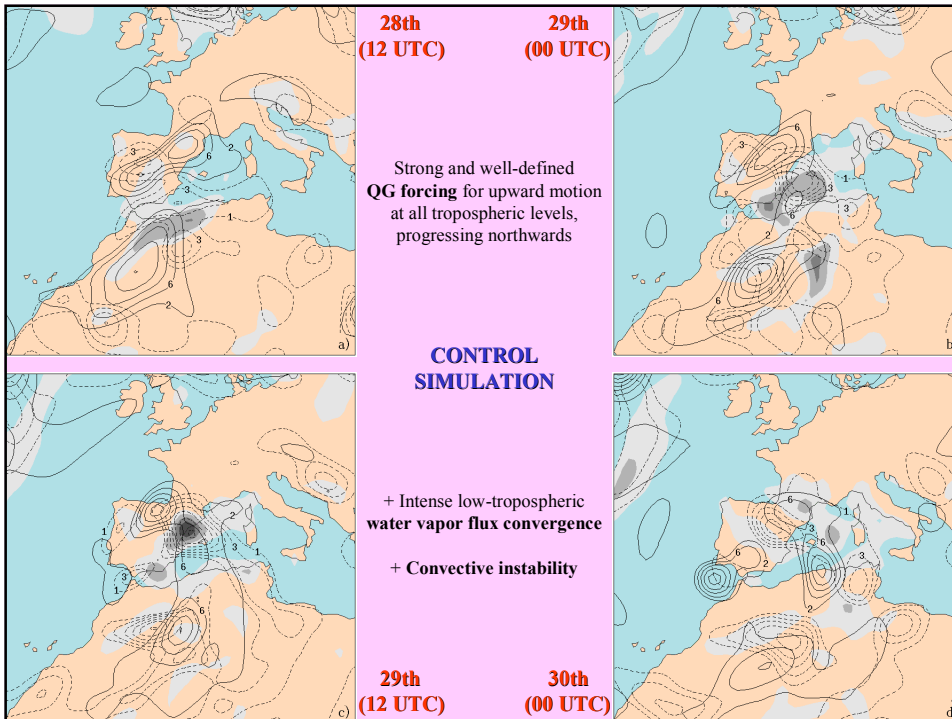
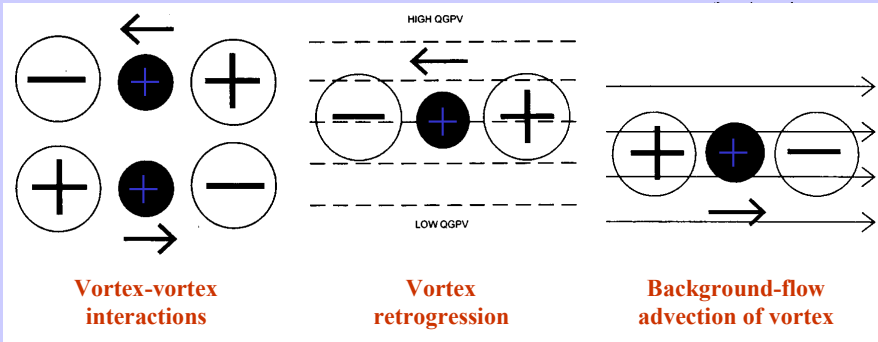
c) About the anomalies:

Same qualitative picture as for the QGPV anomalies





## PV THINKING - Lateral Interactions



## PIECEWISE PV INVERSION TECHNIQUE

(Davis and Emanuel; *MWR* 1991)

1) **Balanced flow** ( $\phi, \psi$ ) given instantaneous distribution of Ertel's PV ( $q$ ):

\* Charney (1955) nonlinear balance equation

$$\nabla^2 \phi = \nabla \cdot f \nabla \psi + 2m^2 \left[ \frac{\partial^2 \psi}{\partial x^2} \frac{\partial^2 \psi}{\partial y^2} - \left( \frac{\partial^2 \psi}{\partial x \partial y} \right)^2 \right]$$

$f$  Coriolis parameter       $m$  map-scale factor

\* Approximate form of Ertel's PV

$$q = \frac{g\kappa\pi}{p} \left[ (f + m^2 \nabla^2 \psi) \frac{\partial^2 \phi}{\partial \pi^2} - m^2 \left( \frac{\partial^2 \psi}{\partial x \partial \pi} \frac{\partial^2 \phi}{\partial x \partial \pi} + \frac{\partial^2 \psi}{\partial y \partial \pi} \frac{\partial^2 \phi}{\partial y \partial \pi} \right) \right]$$

$p$  pressure       $g$  gravity       $\kappa = Rd/C_p$        $\pi = C_p(p/p_0)^\kappa$

\* **Boundary conditions** Lateral (Dirichlet) / Top and Bottom (Neumann):  $\partial \phi / \partial \pi = f \partial \psi / \partial \pi = -\theta$   
 $\theta$  potential temperature

2) **Reference state: Balanced flow** ( $\bar{\phi}, \bar{\psi}$ ) given time mean distribution of Ertel's PV ( $\bar{q}$ ):

\* Same equations as in 1), except using time mean fields instead of instantaneous fields

3) **Perturbation fields** ( $\phi', \psi', q'$ ) given by the definitions:  $(q, \phi, \psi) = (\bar{q}, \bar{\phi}, \bar{\psi}) + (q', \phi', \psi')$

## PIECEWISE PV INVERSION TECHNIQUE

4) We consider that  $q'$  is partitioned into  $N$  portions or anomalies:  $q' = \sum_{n=1}^N q_n$

5) **Piecewise inversion:** ( $\phi_n, \psi_n$ ) associated with  $q_n$ ?

... and requiring:

$$\phi' = \sum_{n=1}^N \phi_n$$

$$\psi' = \sum_{n=1}^N \psi_n$$

...After substitution of the above summations in the balance and PV equations and some rearrangements of the nonlinear terms:

$$\nabla^2 \phi_n = \nabla \cdot f \nabla \psi_n + 2m^2 \left( \frac{\partial^2 \psi^*}{\partial x^2} \frac{\partial^2 \psi_n}{\partial y^2} + \frac{\partial^2 \psi^*}{\partial y^2} \frac{\partial^2 \psi_n}{\partial x^2} - 2 \frac{\partial^2 \psi^*}{\partial x \partial y} \frac{\partial^2 \psi_n}{\partial y \partial x} \right)$$

$$q_n = \frac{g\kappa\pi}{p} \left[ (f + m^2 \nabla^2 \psi^*) \frac{\partial^2 \phi_n}{\partial \pi^2} + m^2 \frac{\partial^2 \phi^*}{\partial \pi^2} \nabla^2 \psi_n - m^2 \left( \frac{\partial^2 \phi^*}{\partial x \partial \pi} \frac{\partial^2 \psi_n}{\partial x \partial \pi} + \frac{\partial^2 \phi^*}{\partial y \partial \pi} \frac{\partial^2 \psi_n}{\partial y \partial \pi} \right) - m^2 \left( \frac{\partial^2 \psi^*}{\partial x \partial \pi} \frac{\partial^2 \phi_n}{\partial x \partial \pi} + \frac{\partial^2 \psi^*}{\partial y \partial \pi} \frac{\partial^2 \phi_n}{\partial y \partial \pi} \right) \right]$$

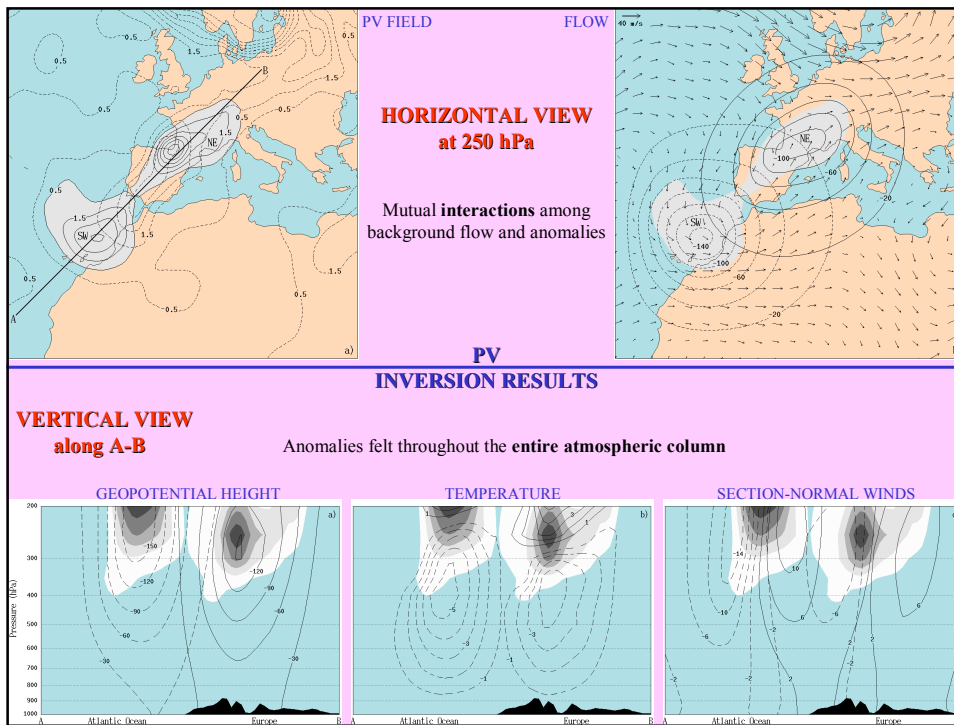
where  $(\ )^* = \bar{(\ )} + \frac{1}{2}(\ )'$

**Boundary conditions:** Lateral (homogeneous) / Top and bottom (using  $\theta_n$ )

At 00 UTC 28 September 1994, using the NCEP-based isobaric analysis

\* **In our case study:** Reference state: 6-day time average about 00 UTC 28 September

**Anomalies:** positive PV perturbations above 500 hPa **SW** and **NE** of Gulf of Cádiz



### SENSITIVITY EXPERIMENTS

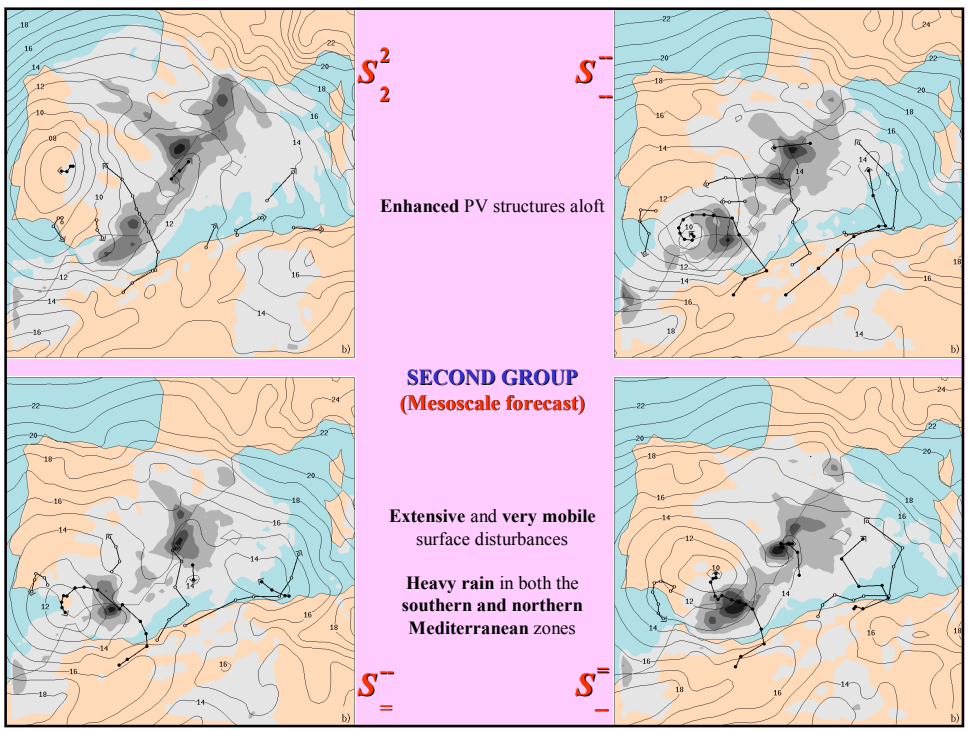
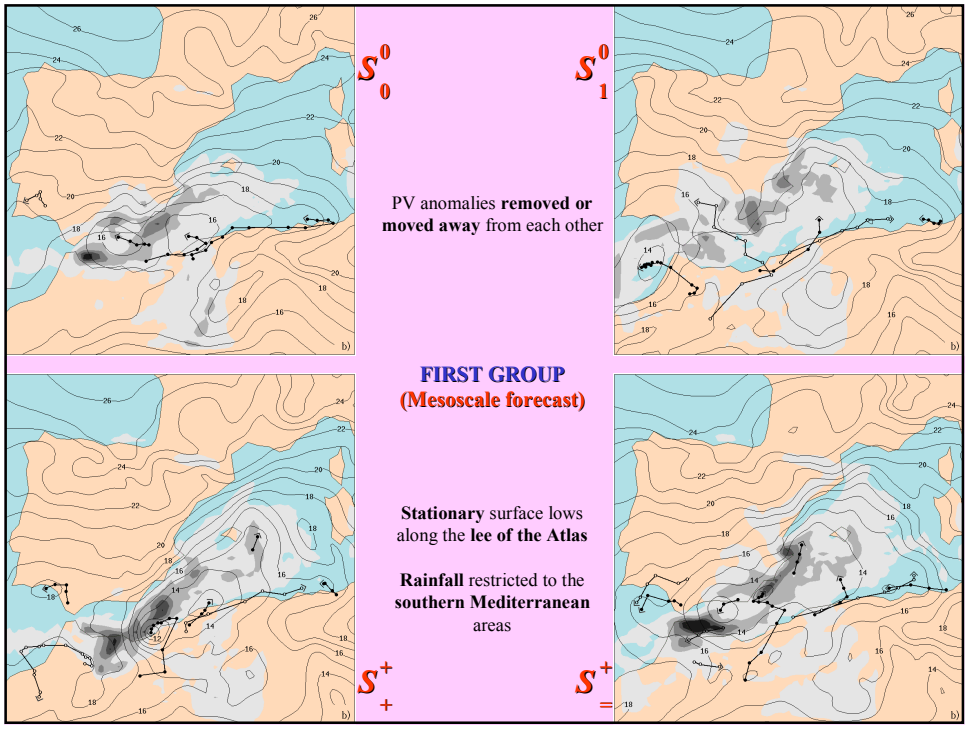
**By adding and/or subtracting the PV-inverted balanced fields (geopotential, temperature and wind) into the model initial conditions**

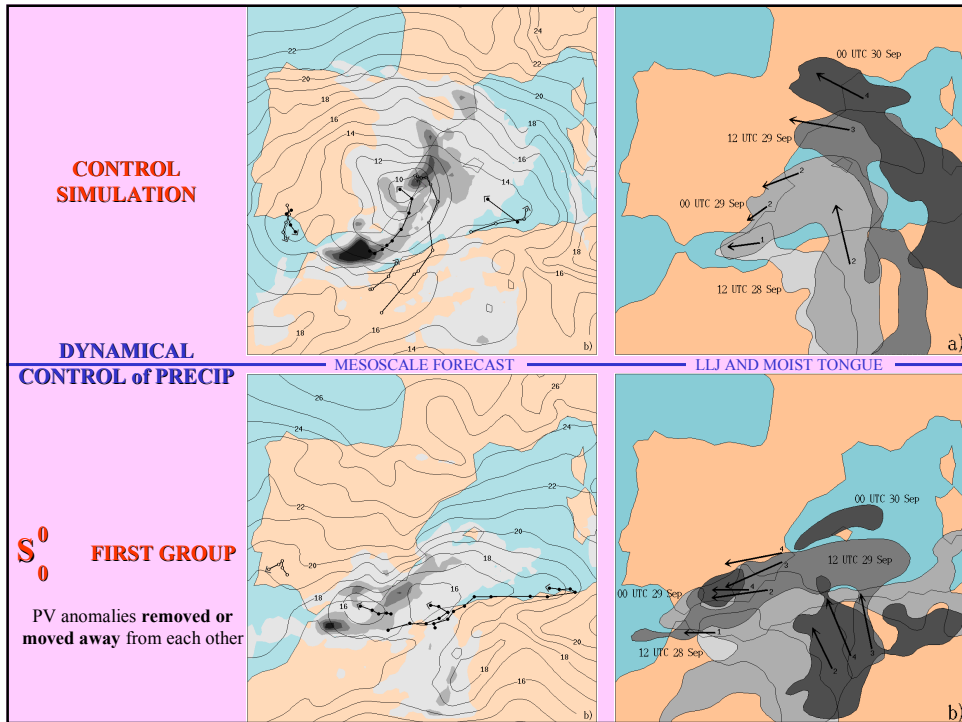
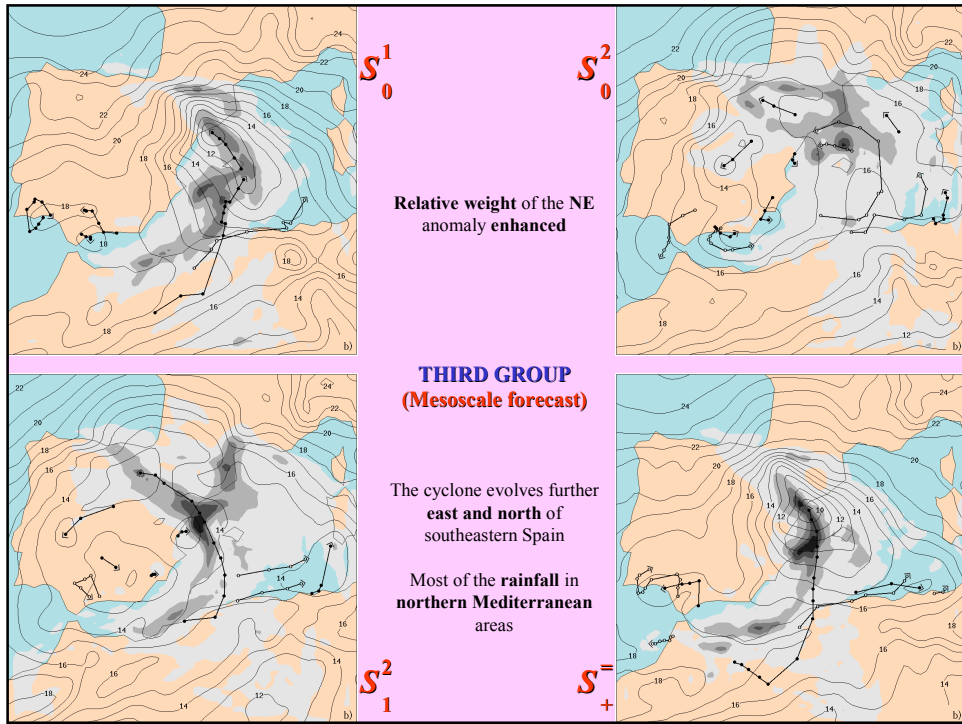
**Sensitivity to the intensity**  
(One or both PV anomalies removed or doubled)

Experiment	SW anomaly	NE anomaly
$S_0^0$	Removed	Removed
$S_2^2$	Doubled	Doubled
$S_1^0$	Unchanged	Removed
$S_0^2$	Doubled	Removed
$S_0^1$	Removed	Unchanged
$S_0^2$	Removed	Doubled
$S_2^1$	Doubled	Unchanged
$S_1^2$	Unchanged	Doubled

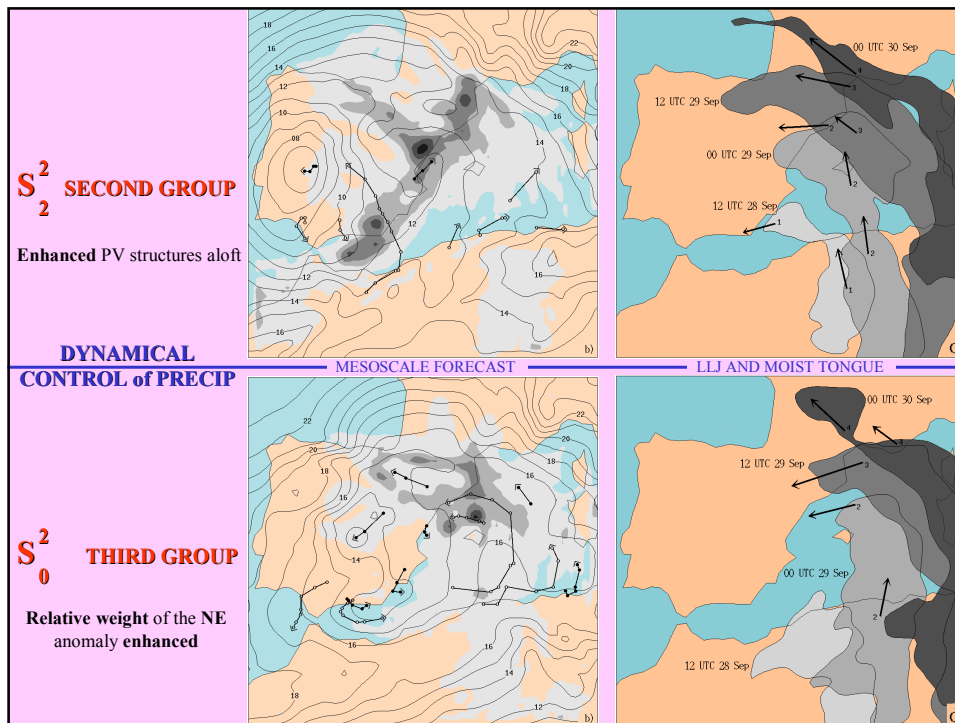
**Sensitivity to the position**  
(One or both PV anomalies shifted 425 km along A-B)

Experiment	SW anomaly	NE anomaly
$S_-^-$	Moved inwards	Moved inwards
$S_+^+$	Moved outwards	Moved outwards
$S_-^-$	Unchanged	Moved inwards
$S_+^-$	Moved outwards	Moved inwards
$S_-^=$	Moved inwards	Unchanged
$S_+^+$	Moved inwards	Moved outwards
$S_+^-$	Moved outwards	Unchanged
$S_-^=$	Unchanged	Moved outwards









### INTRODUCTION - PART 3

\* The previous example shows that the two embedded **upper-level PV centres** played an **important role** for the evolution, intensity and spatial extent of the **surface cyclone**

\* How a potential analysis and/or forecast **error** in the representation of the precursor **upper-level trough** would affect a **mesoscale forecast** ?



\* Such uncertainties can be accounted for by means of an **ensemble prediction system** defined by a collection of simulations with perturbed initial conditions (using the PV inversion method)

\* **How much** to perturb? A **PV-error climatology** has been derived

\* **Where** to perturb? Sensitivity areas according to **MM5-adjoint** run / **human-based** criteria

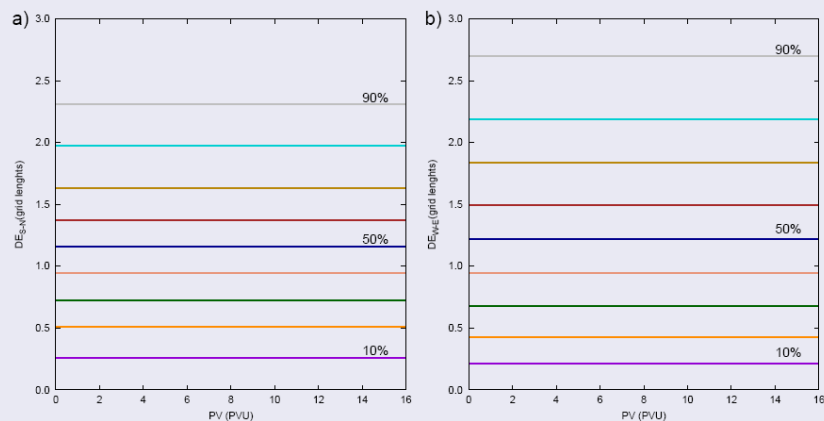
## PV error climatology

Comparing the PV fields of  
ECMWF **analysis**  $\longleftrightarrow$  ECMWF **24 h forecast**,  
of a large collection of MEDEX cyclones,  
one can define:

- The **displacement error** (DE): the minimum displacement of the 24 h forecast PV field showing local maximum correlation with the analysis PV field
- The **intensity error** (IE): the difference between the displaced 24 h forecast PV field and analysis PV field relative to the analysis PV average

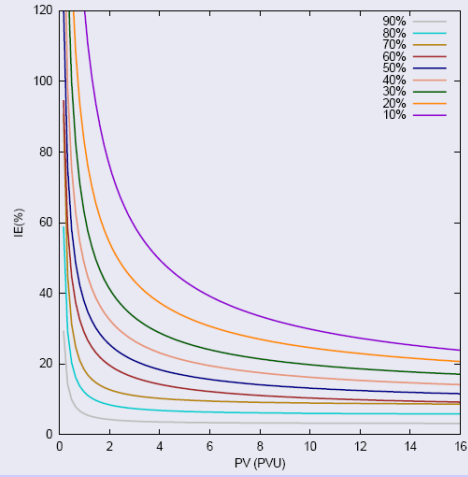
## PV error climatology: Percentile levels at 300 hPa

### Displacement Error



### PV error climatology: Percentile levels at 300 hPa

Intensity Error

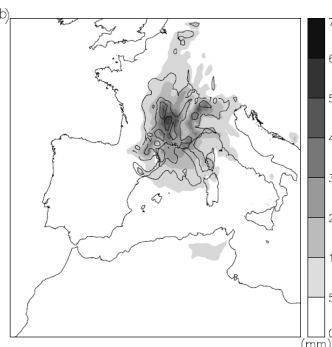
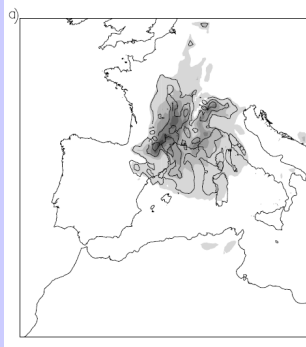
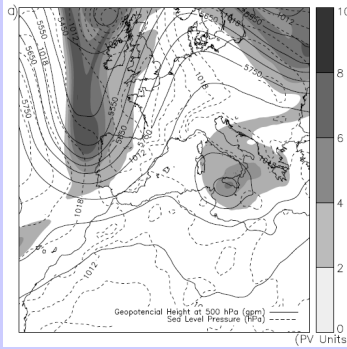


### EXAMPLE

9 June 2000 at 00 UTC

PV-gradient

PV-adjoint



## Results

- The two ensembles have a good performance (better than a multiphysics EPS)
- PV-gradient performs better than PV-adjoint
- PV-adjoint higher computational cost than the PV-gradient

## INTRODUCTION - PART 4

### LIFE CYCLE OF AN INTENSE MEDITERRANEAN CYCLONE

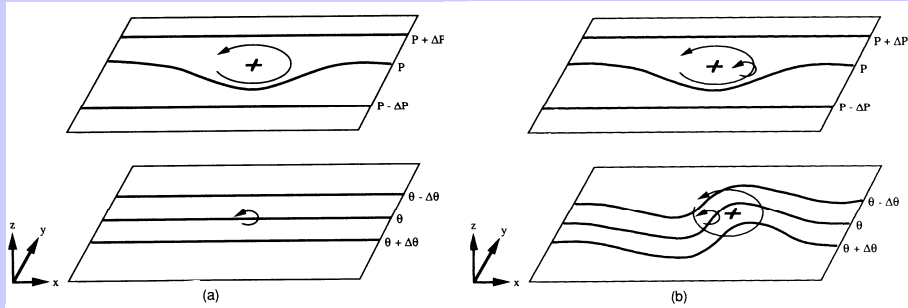
**PV THINKING** → An analysis of the cyclone event in terms of the **impacts** and **interactions** of dry and moist **PV anomalies** (and mean flow)

Beyond a qualitative analysis, **how** can these impacts and interactions be **quantified ???**



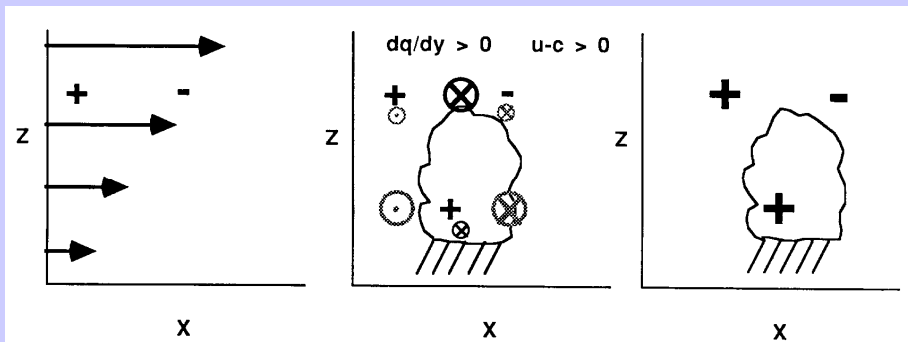
**PV-BASED PROGNOSTIC SYSTEM + FACTOR SEPARATION**  
(without the need of numerical simulations !!!)

### PV THINKING - Vertical Interactions



Growth of an idealized baroclinic wave-cyclone

### PV THINKING - Vertical Interactions



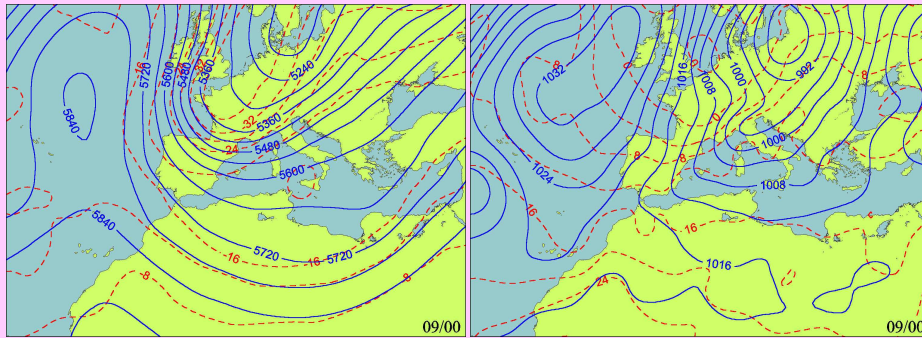
Effects of diabatic processes (condensation)



*LIFE CYCLE OF THE CYCLONE (9-12 November 2001)*

Mid-Upper levels ( H 500 / T 500)

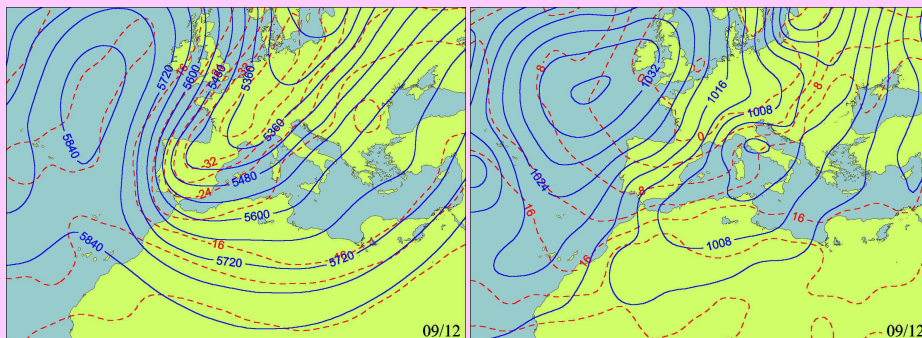
Low levels (SLP / T 925)



*LIFE CYCLE OF THE CYCLONE (9-12 November 2001)*

Mid-Upper levels ( H 500 / T 500)

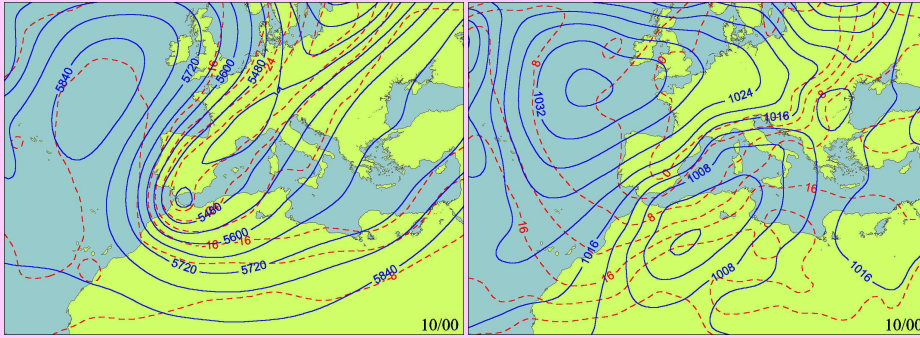
Low levels (SLP / T 925)



*LIFE CYCLE OF THE CYCLONE (9-12 November 2001)*

Mid-Upper levels ( H 500 / T 500 )

Low levels ( SLP / T 925 )



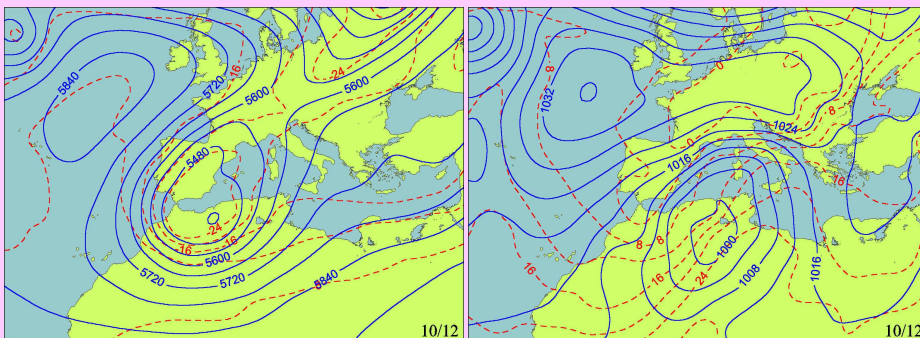
**ALGERIA**

- Over 100 mm/6 h that led to catastrophic flooding
- 737 people were killed and 23000 left homeless

*LIFE CYCLE OF THE CYCLONE (9-12 November 2001)*

Mid-Upper levels ( H 500 / T 500 )

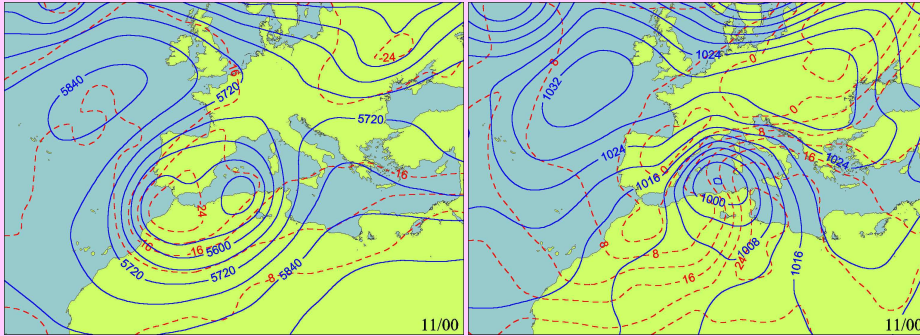
Low levels ( SLP / T 925 )



*LIFE CYCLE OF THE CYCLONE (9-12 November 2001)*

Mid-Upper levels ( H 500 / T 500)

Low levels (SLP / T 925)



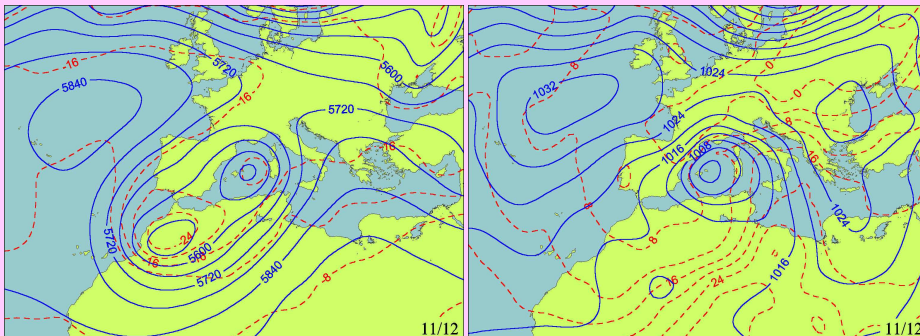
**BALEARIC ISLANDS**

- Up to 400 mm/24 h, 150 km/h winds and 12 m sea waves
- 4 casualties, 500000 trees uprooted, floods and severe damages on coasts

*LIFE CYCLE OF THE CYCLONE (9-12 November 2001)*

Mid-Upper levels ( H 500 / T 500)

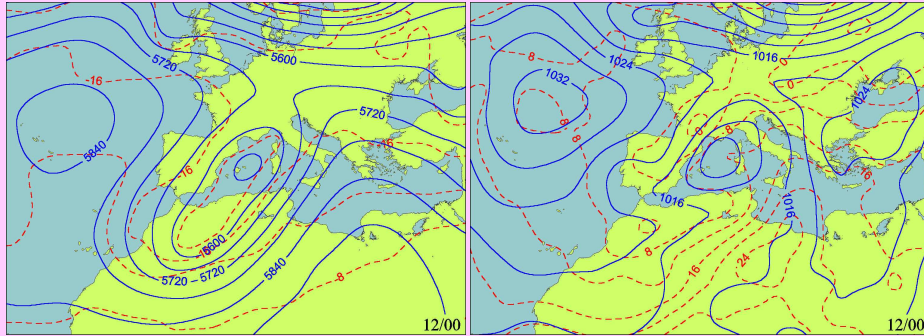
Low levels (SLP / T 925)



*LIFE CYCLE OF THE CYCLONE (9-12 November 2001)*

Mid-Upper levels ( H 500 / T 500)

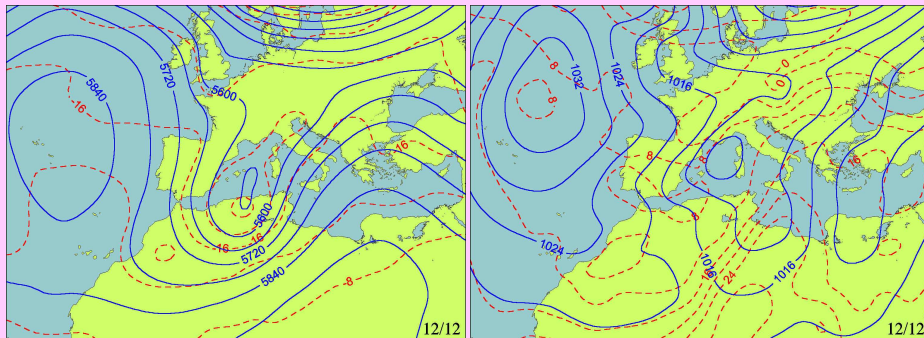
Low levels (SLP / T 925)



*LIFE CYCLE OF THE CYCLONE (9-12 November 2001)*

Mid-Upper levels ( H 500 / T 500)

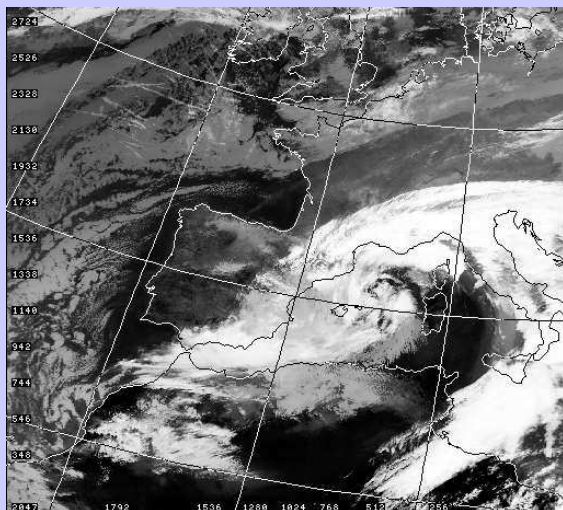
Low levels (SLP / T 925)



Strong baroclinic  
development



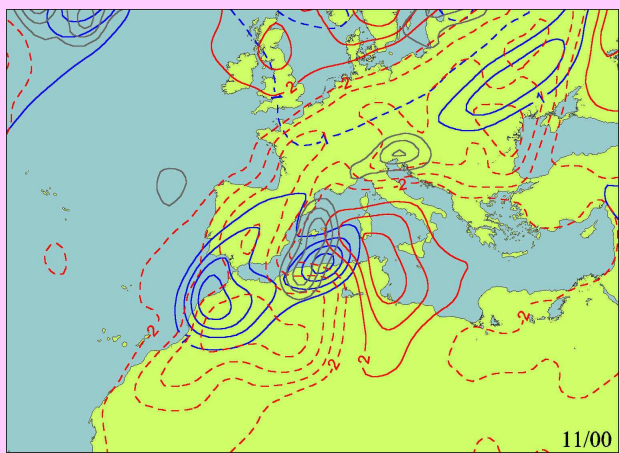
Ch4-IR NOAA image (11 Nov / 13.29 UTC)



Diabatic contribution ?

*PV-based  
DIAGNOSIS*

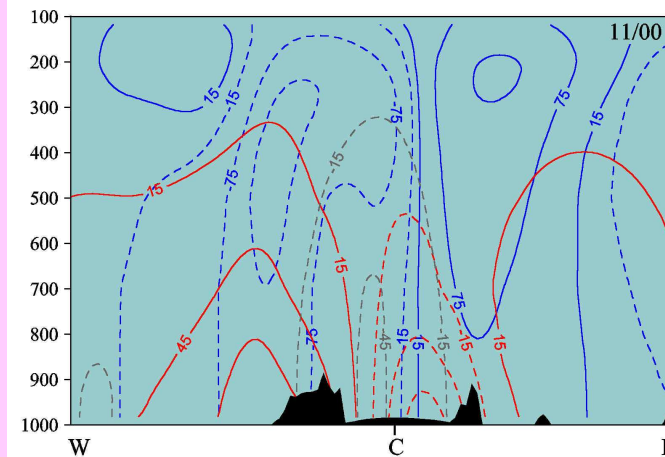
<b>ULev</b>	PV perturbation above 700 hPa
<b>LLev</b>	Surface thermal anomaly and PV perturbation below 700 hPa
<b>DIAB</b>	Positive PV perturbation below 500 hPa in areas with RH > 70%



*PV-based  
DIAGNOSIS*

<b>ULev</b>	<b>PV perturbation above 700 hPa</b>
<b>LLev</b>	<b>Surface thermal anomaly and PV perturbation below 700 hPa</b>
<b>DIAB</b>	<b>Positive PV perturbation below 500 hPa in areas with RH &gt; 70%</b>

Geopotential height perturbation



**PV-BASED PROGNOSTIC SYSTEM  
(Davis and Emanuel; MWR 1991)**

0) A balanced flow has been first found using the PV inversion technique:  $q \rightarrow (\phi, \psi)$

1) Tendency of the Charney (1955) nonlinear balance equation:

$$\nabla^2 \phi^t = \nabla \cdot f \nabla \psi^t + 2m^2 \left[ \frac{\partial^2 \psi^t \partial^2 \psi}{\partial x^2 \partial y^2} + \frac{\partial^2 \psi \partial^2 \psi^t}{\partial x^2 \partial y^2} - 2 \frac{\partial^2 \psi \partial^2 \psi^t}{\partial x \partial y \partial x \partial y} \right]$$

2) Tendency of the approximate form of Ertel's PV:

$$q^t = \frac{g\kappa\pi}{p} \left[ (f + m^2 \nabla^2 \psi) \frac{\partial^2 \phi^t}{\partial \pi^2} + m^2 \frac{\partial^2 \phi}{\partial \pi^2} \nabla^2 \psi^t - m^2 \left( \frac{\partial^2 \psi^t \partial^2 \phi}{\partial x \partial \pi \partial x \partial \pi} + \frac{\partial^2 \psi \partial^2 \phi^t}{\partial x \partial \pi \partial x \partial \pi} + \frac{\partial^2 \psi^t \partial^2 \phi}{\partial y \partial \pi \partial y \partial \pi} + \frac{\partial^2 \psi \partial^2 \phi^t}{\partial y \partial \pi \partial y \partial \pi} \right) \right]$$

$(\phi^t, \psi^t)$

3) Ertel's PV tendency equation (frictionless but with diabatic term included):

$$q^t = -m(\mathbf{V}_\psi + \mathbf{V}_\chi) \cdot \nabla q - \omega^* \frac{\partial q}{\partial \pi} + \frac{m}{\rho} \boldsymbol{\eta} \cdot \nabla LH$$

$\mathbf{V}_\psi = m\mathbf{k} \times \nabla \psi$   
 $\mathbf{V}_\chi = m\nabla \chi$

$\omega^* = \frac{d\pi}{dt} = \frac{\kappa\pi}{p} \omega$

$q^t$

## PV-BASED PROGNOSTIC SYSTEM

4) Omega equation:

$$\begin{aligned}
 & f\eta \frac{\partial}{\partial \pi} \left[ \pi^{1-1/\kappa} \frac{\partial}{\partial \pi} (\pi^{1/\kappa-1} \omega^*) \right] + m^2 \nabla^2 \left( \frac{\partial^2 \phi}{\partial \pi^2} \omega^* \right) \\
 & - m^2 f \frac{\partial}{\partial \pi} \left( \frac{\partial \omega^*}{\partial x} \frac{\partial \psi}{\partial x \partial \pi} + \frac{\partial \omega^*}{\partial y} \frac{\partial \psi}{\partial y \partial \pi} \right) \\
 & + \left( f \frac{\partial \eta}{\partial \pi} \frac{1/\kappa - 1}{\pi} - f \frac{\partial^2 \eta}{\partial \pi^2} \right) \omega^* = m^3 \nabla^2 [(\mathbf{V}_\psi + \mathbf{V}_\chi) \cdot \nabla \theta] \longrightarrow \omega^* \\
 & + m f \frac{\partial}{\partial \pi} [(\mathbf{V}_\psi + \mathbf{V}_\chi) \cdot \nabla \eta] - m^2 \nabla f \cdot \nabla \left( \frac{\partial \psi^t}{\partial \pi} \right) \\
 & - 2m^4 \frac{\partial}{\partial \pi} \left[ \frac{\partial^2 \psi^t}{\partial x^2} \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial x^2} \frac{\partial^2 \psi^t}{\partial y^2} - 2 \frac{\partial^2 \psi}{\partial x \partial y} \frac{\partial^2 \psi^t}{\partial x \partial y} \right] \\
 & - m^2 \nabla^2 LH
 \end{aligned}$$

5) Continuity equation:

$$m^2 \nabla^2 \chi + \pi^{1-1/\kappa} \frac{\partial}{\partial \pi} (\pi^{1/\kappa-1} \omega^*) = 0 \longrightarrow \chi$$

Lateral B.C (Homogeneous)    Top-Bottom B.C (Neumann)

$$\phi^t = \psi^t = q^t = \omega^* = \chi = 0$$

$$\partial \phi^t / \partial \pi = f \partial \psi^t / \partial \pi = -\theta^t$$

$$\begin{aligned}
 \theta^t &= -m(\mathbf{V}_\psi + \mathbf{V}_\chi) \cdot \nabla \theta - \omega^* \frac{\partial \theta}{\partial \pi} \\
 &+ LH
 \end{aligned}$$

$$\omega_T^* = 0 \quad \omega_B^* = \text{Topographic}$$

## FACTOR SEPARATION (Stein and Alpert, JAS 1993)

0: MEAN + 3 FACTORS (1: ULev 2: LLev 3: DIAB)

$$E_0 = F_0$$

$$E_1 = F_1 - F_0$$

$$E_2 = F_2 - F_0$$

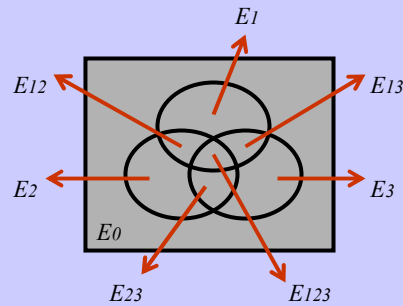
$$E_3 = F_3 - F_0$$

$$E_{12} = F_{12} - (F_1 + F_2) + F_0$$

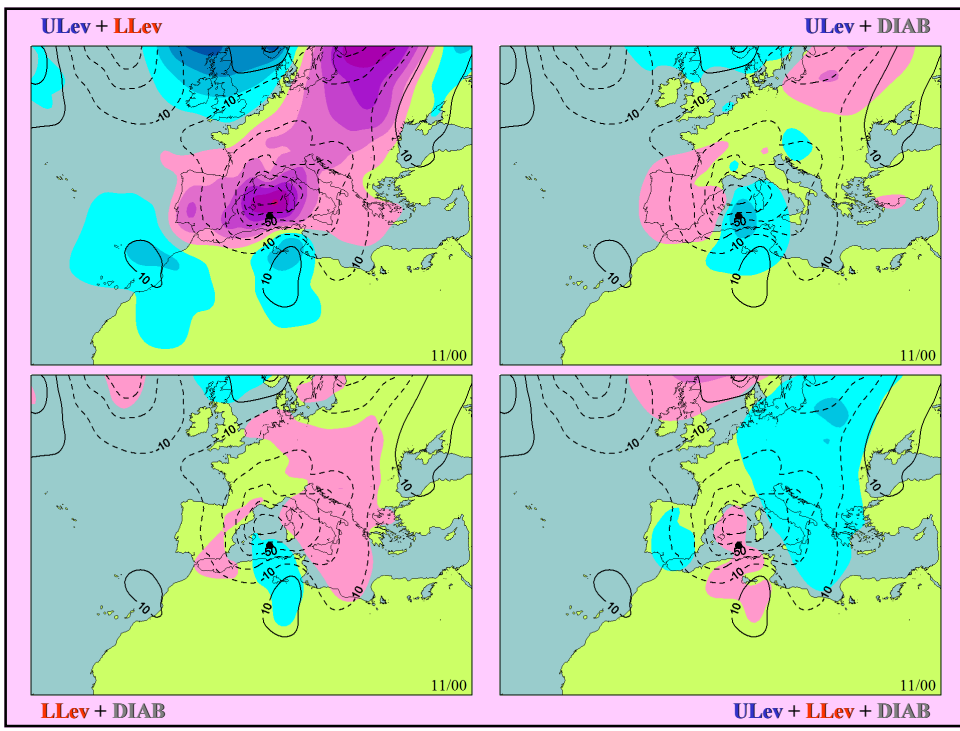
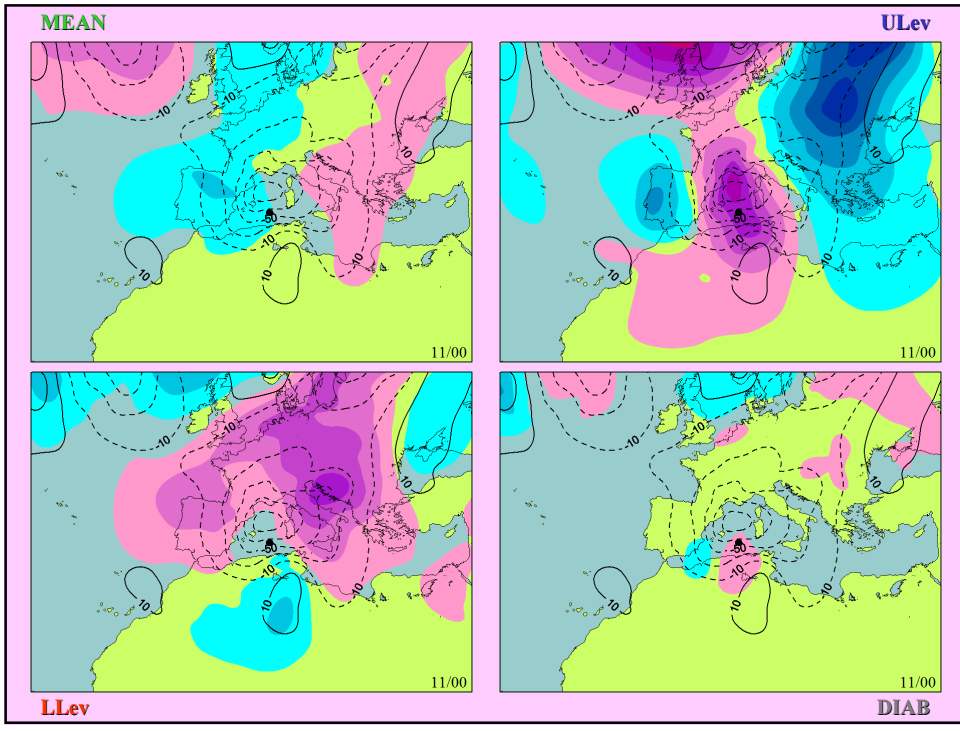
$$E_{13} = F_{13} - (F_1 + F_3) + F_0$$

$$E_{23} = F_{23} - (F_2 + F_3) + F_0$$

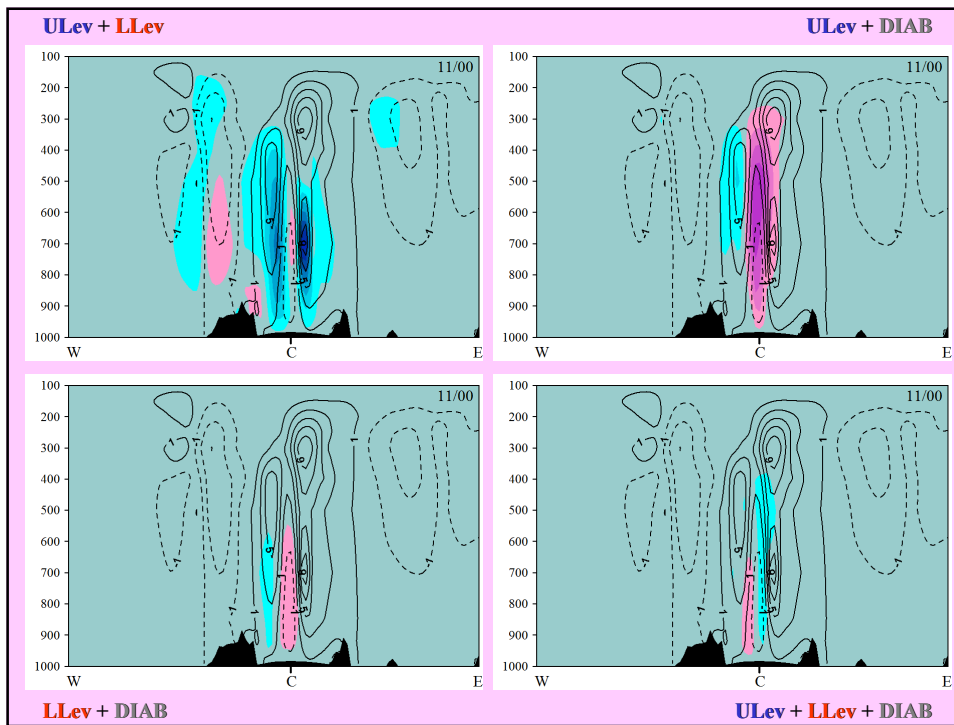
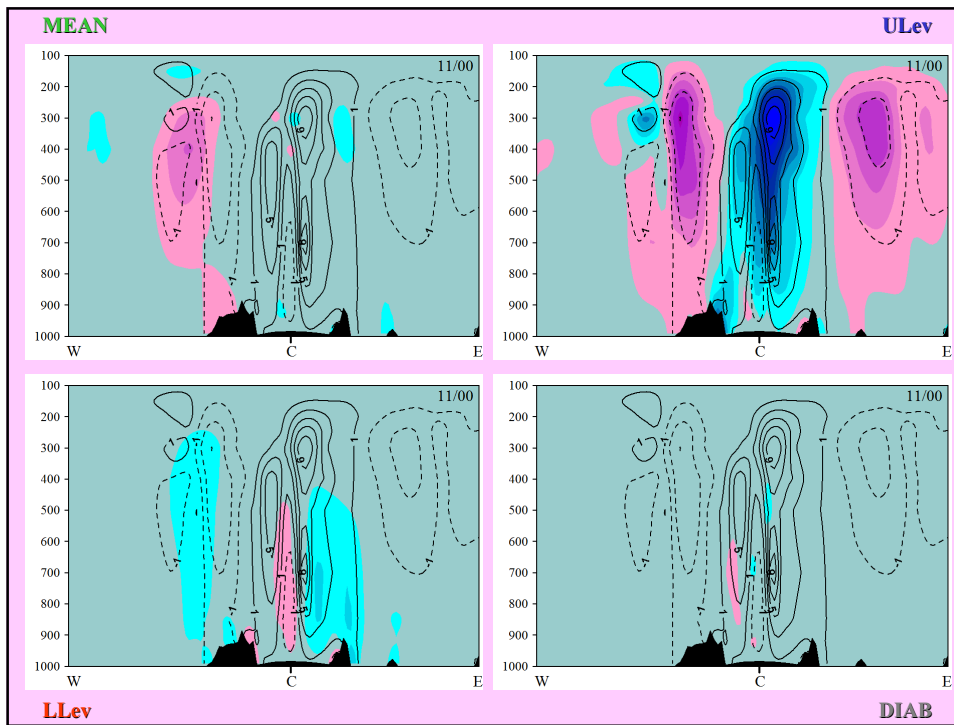
$$E_{123} = F_{123} - (F_{12} + F_{13} + F_{23}) + (F_1 + F_2 + F_3) - F_0$$



( 8 flow configurations necessary )







**THE END !!!**